Dynamical models: from neural microcircuits to cortical regions

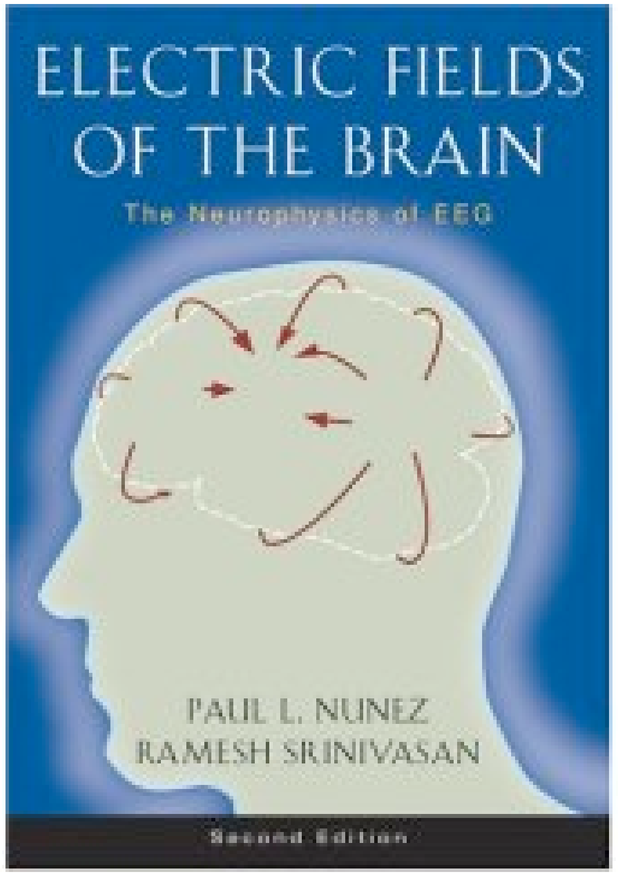
Steve Coombes

The University of Nottingham

S Coombes 2005 Waves, bumps, and patterns in neural field theories, Biological Cybernetics, Vol 93, 91-108.
http://eprints.nottingham.ac.uk/153/

http://www.scholarpedia.org/article/Neural_fields

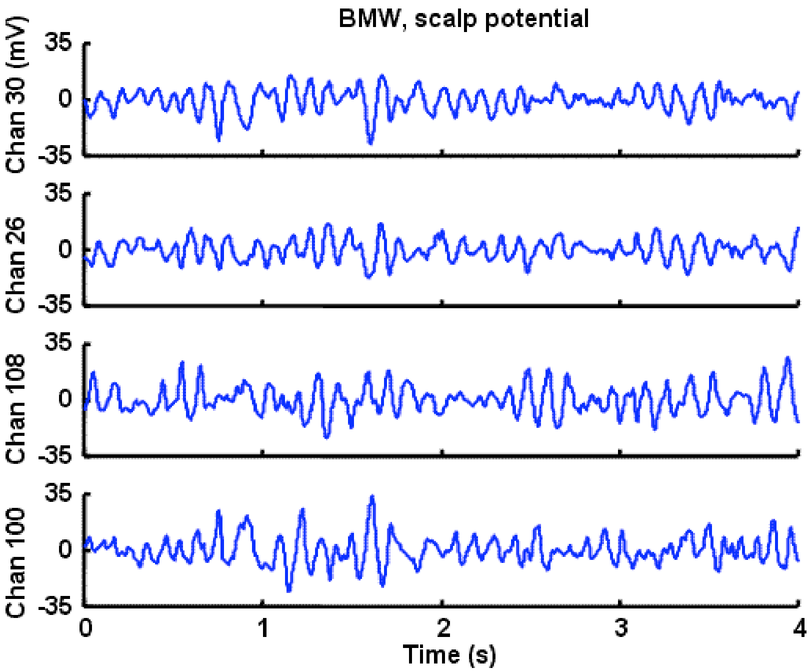
How do we get to a brain-wave equation?



Electric Fields of the Brain: The Neurophysics of EEG

Paul L. Nunez and Ramesh Srinivasan, OUP 2006

$$\left(1+\frac{1}{\alpha}\frac{\partial}{\partial t}\right)u=\psi$$



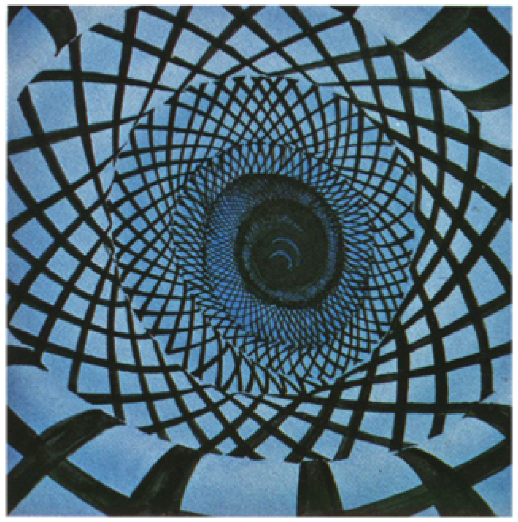
EEG Rhythms

$$[(v+\partial_t)^2 - v^2\partial_{xx}]\psi(x,t) = [v^2 + v\partial_t]f \circ u(x,t)$$

V.K. Jirsa, H. Haken. A Derivation of a Macroscopic Field Theory of the Brain from the Quasi microscopic Neural Dynamics, Physica D 99, 503-526 (1997)

What else is it good for?

Visual hallucinations
(Ermentrout & Cowan)



P. C. Bressloff, J. D. Cowan, M. Golubitsky, P. J. Thomas and M. Wiener Geometric visual hallucinations, Euclidean symmetry and the functional architecture of striate cortex Phil. Trans. R. Soc. B 40:299-330 (2001).

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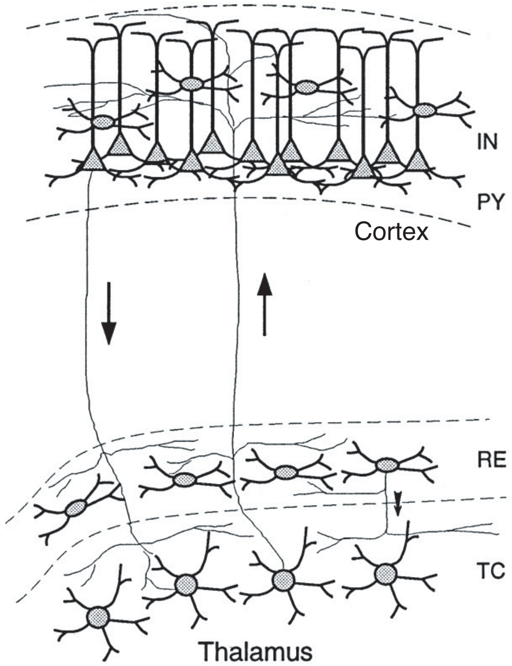
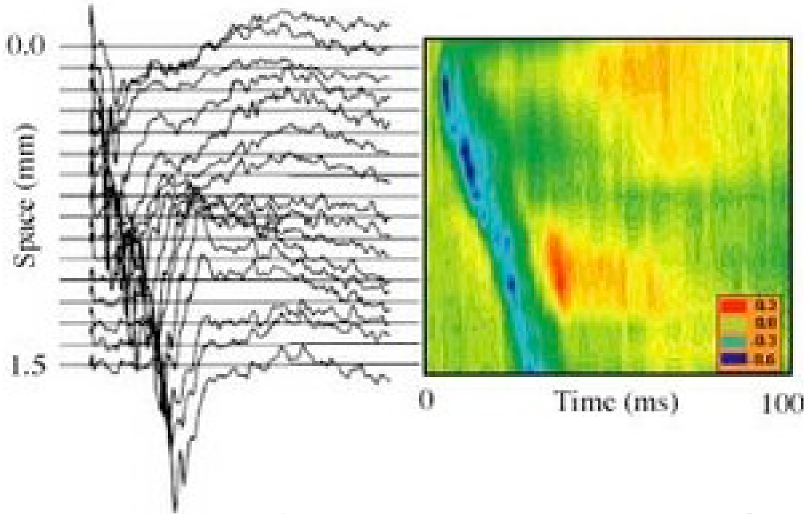
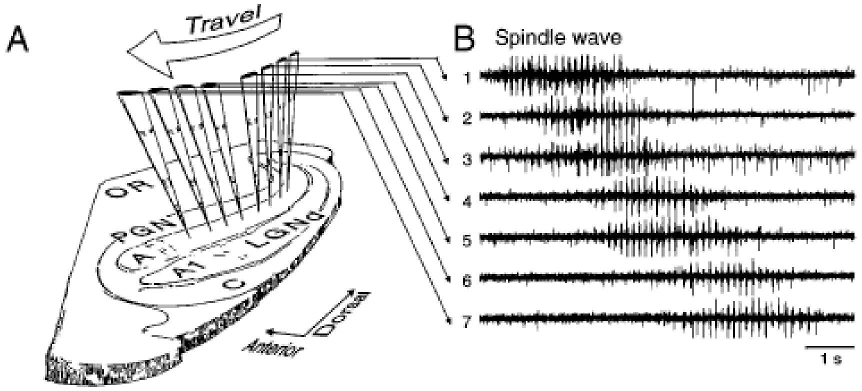




Cortical waves cm/s



Spindle waves in thalamic slices mm/s

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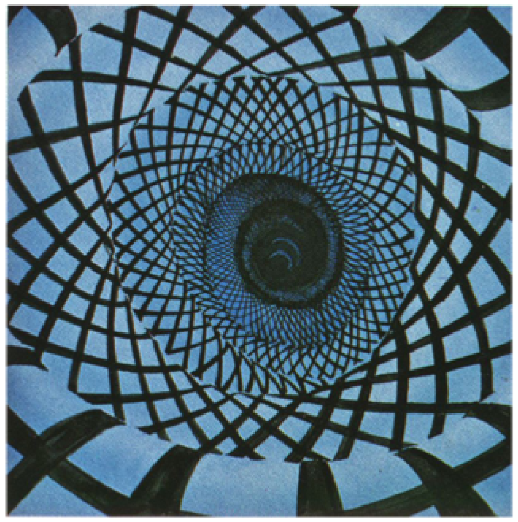

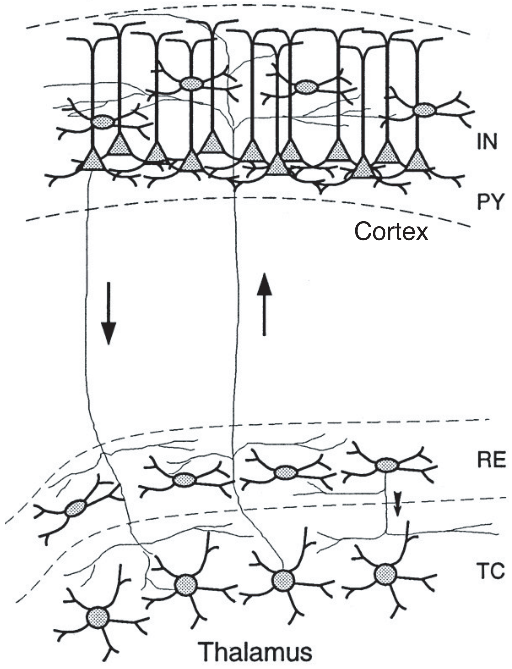

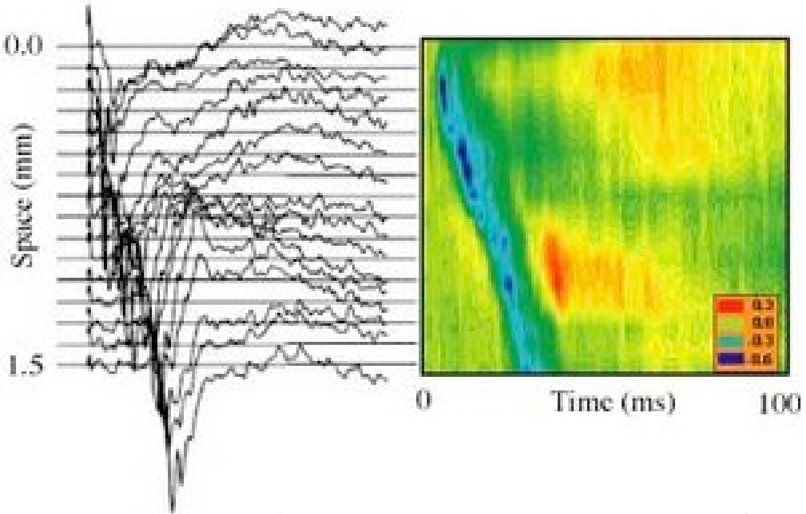

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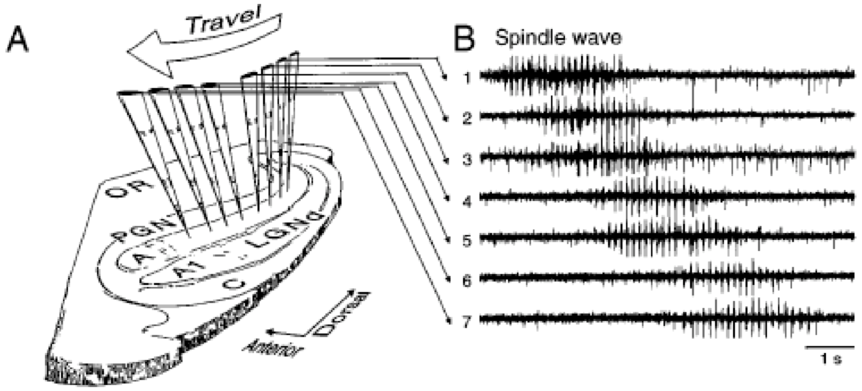

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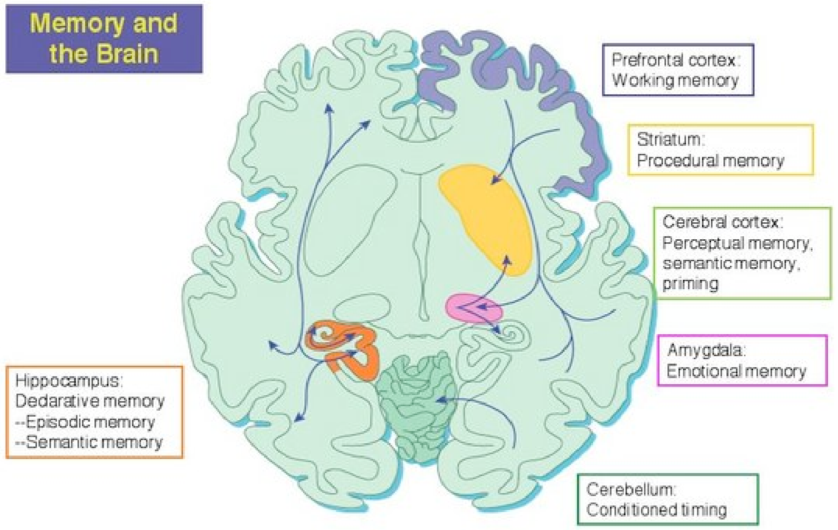
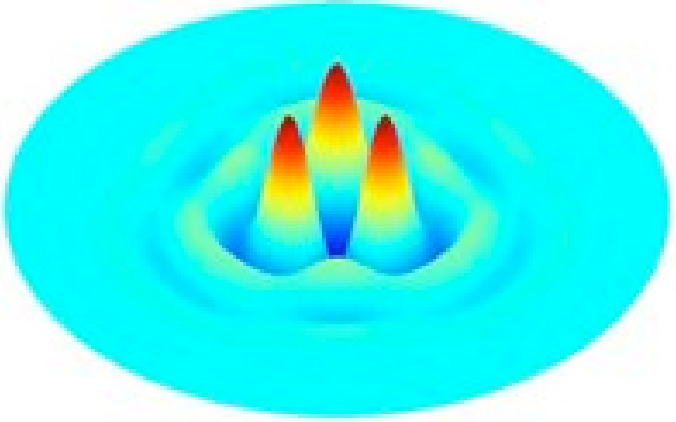


mechanisms for short term memory



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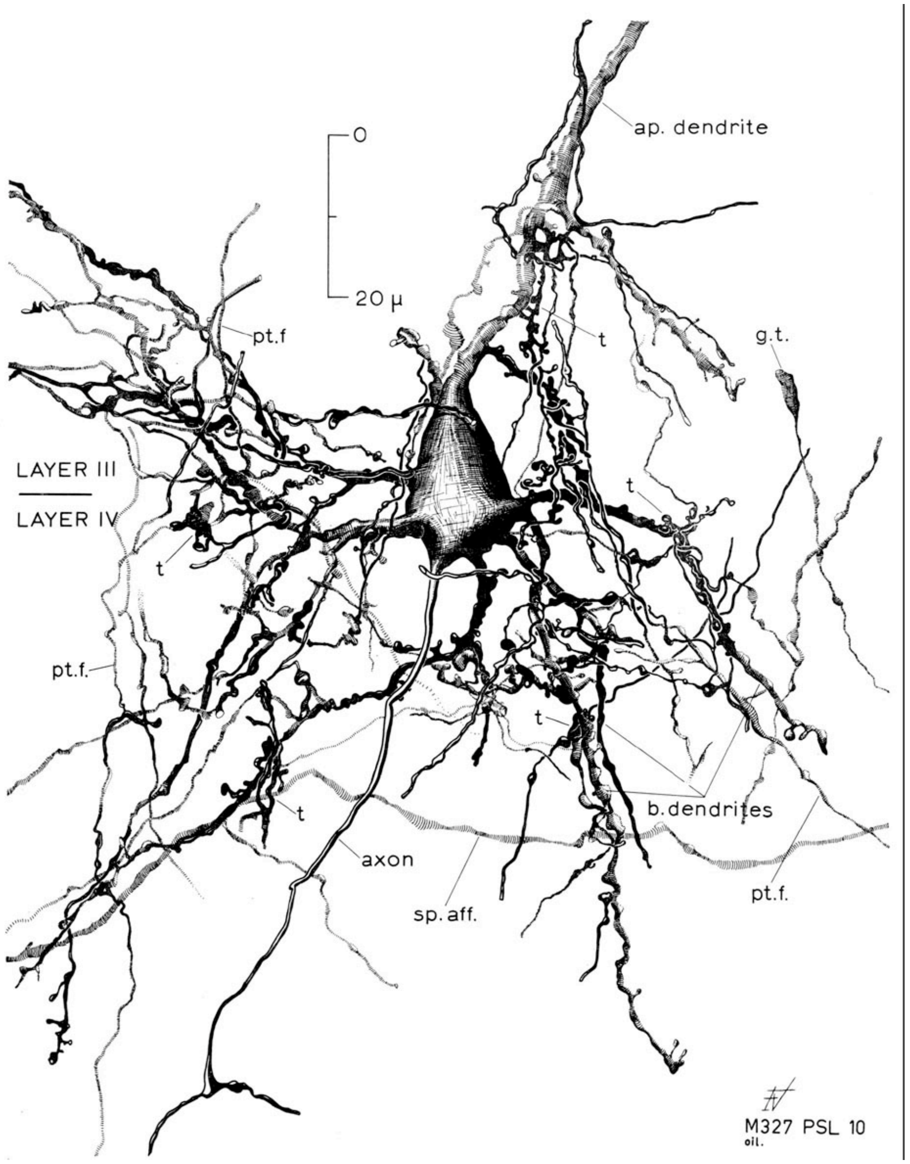
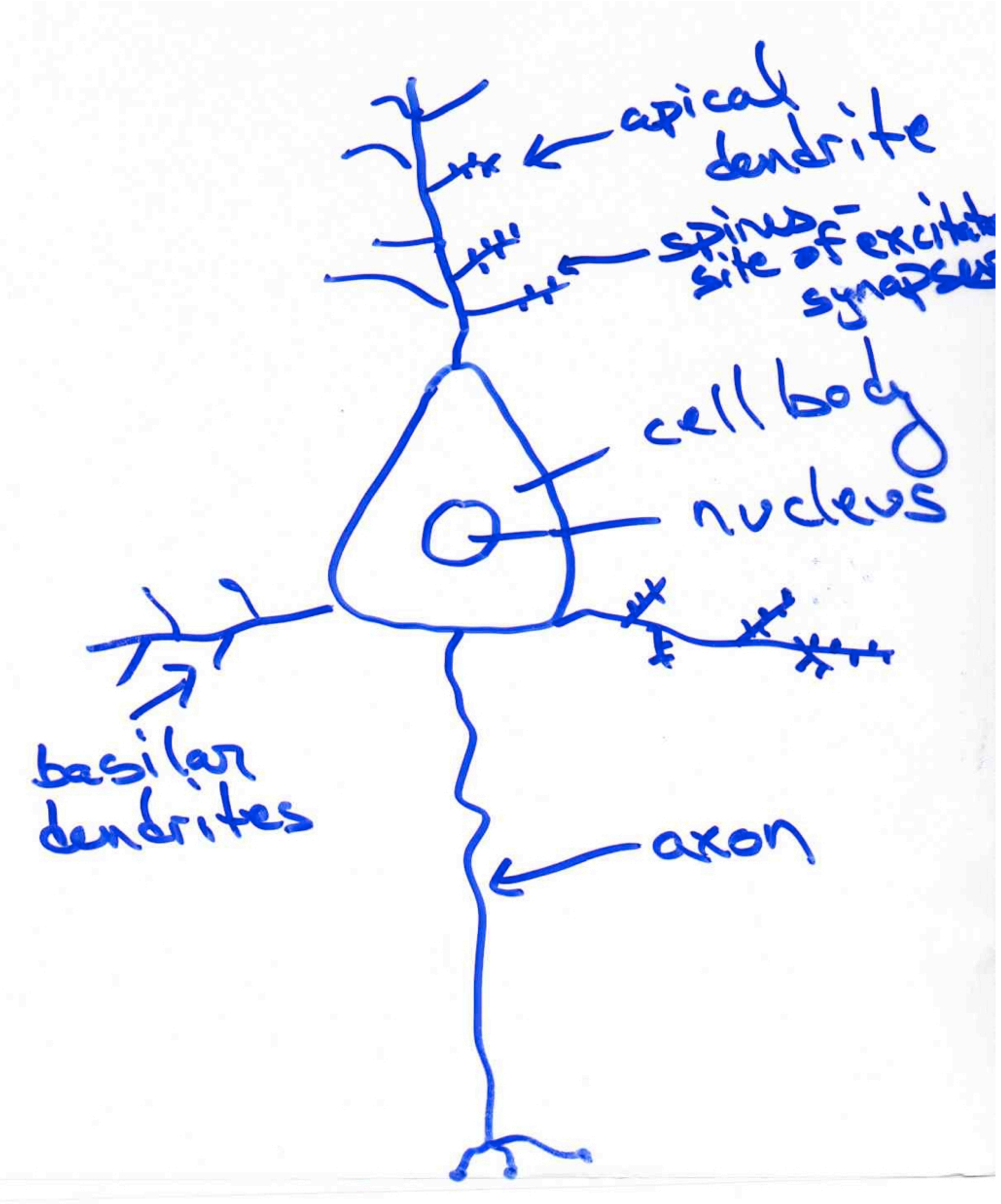
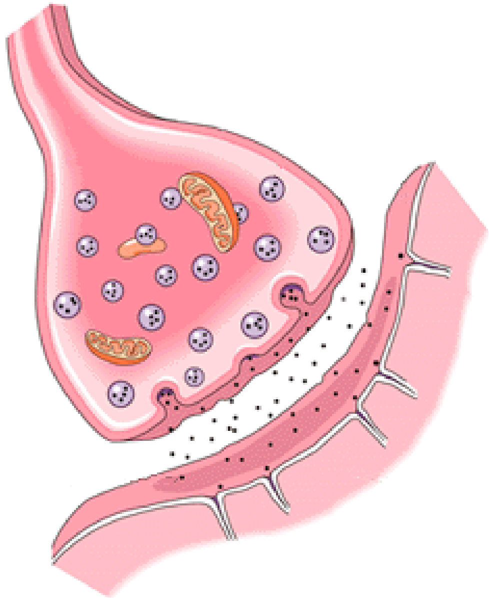
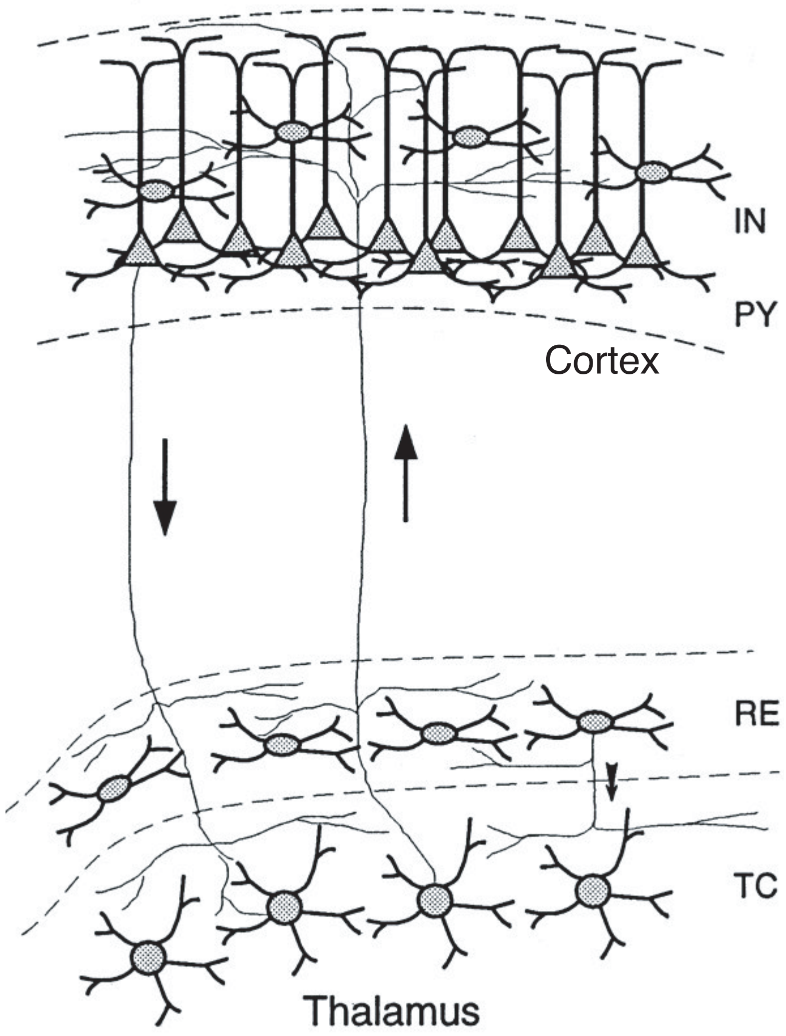
Neurons: pyramidal cells



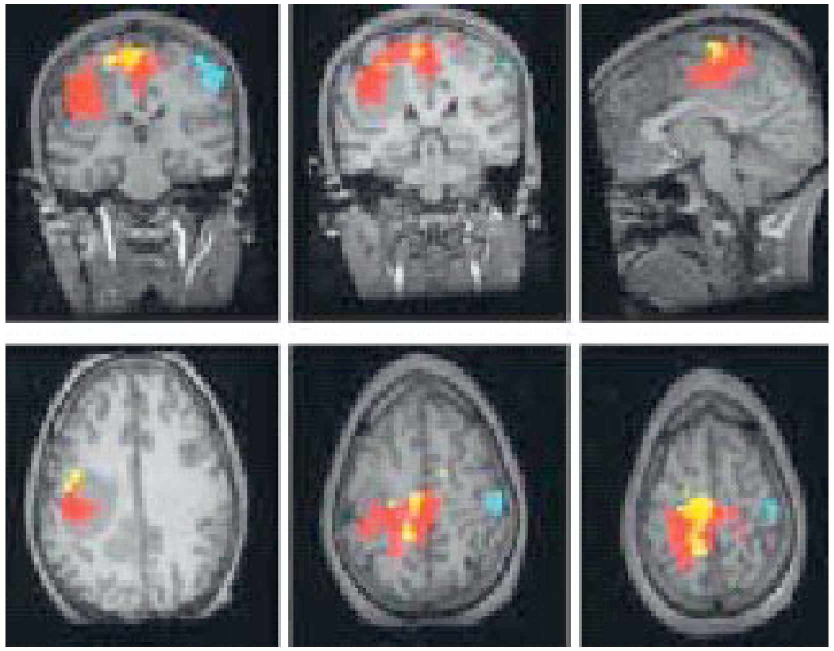


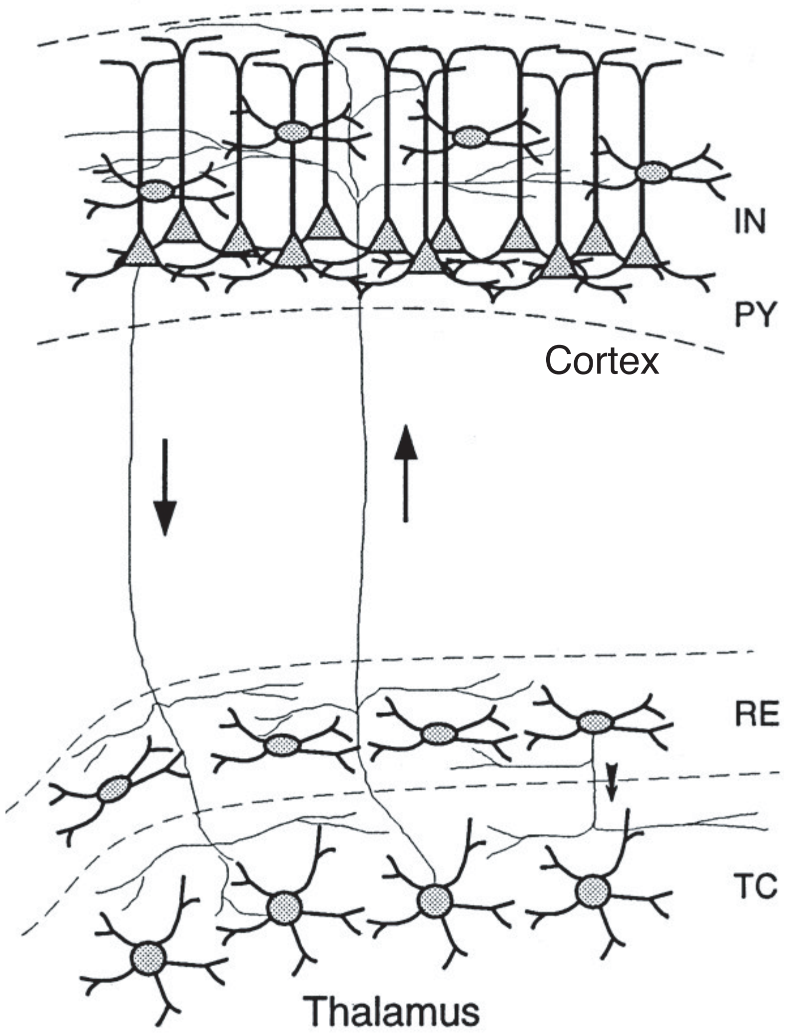




need a spiking single neuron model

HH, FHN, ML, WB, PR, ..., IF





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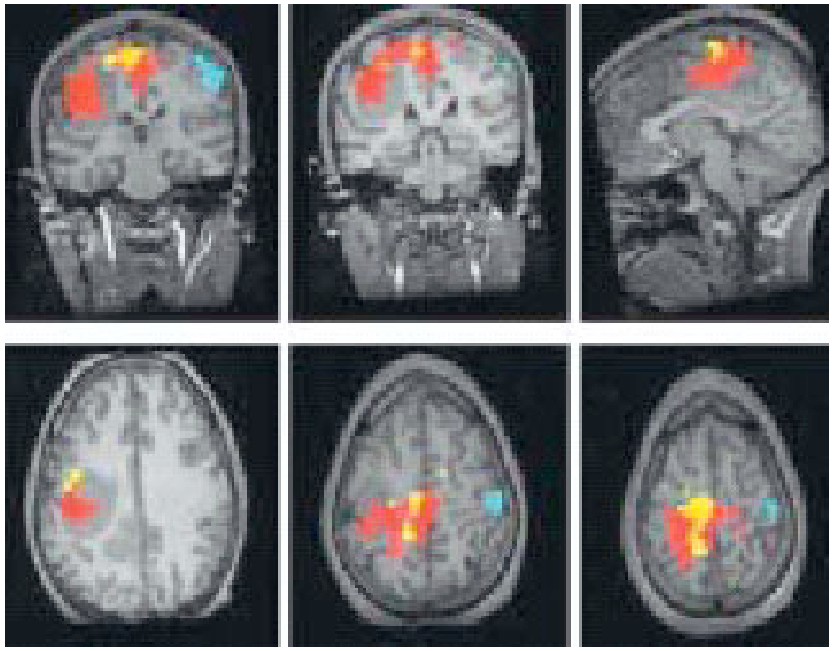
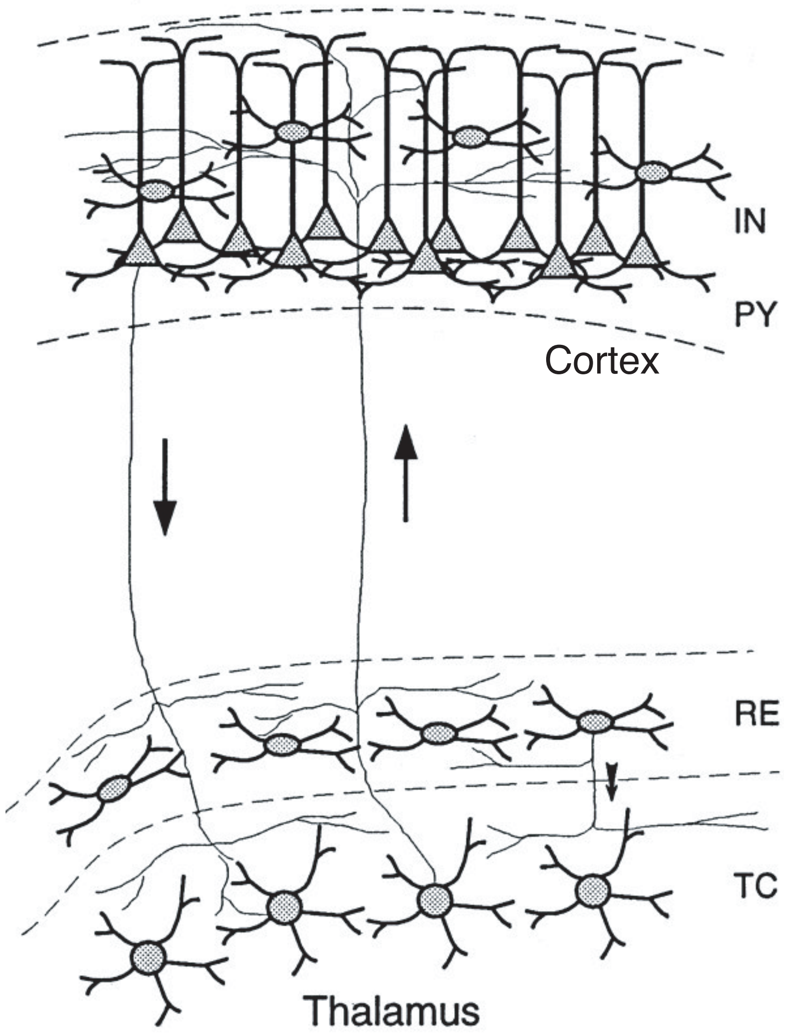
$$C\frac{\mathrm{d}v}{\mathrm{d}t} = -\sum_k g_k m_k^{p_k} h_k^{q_k}(v - v_k) + u$$





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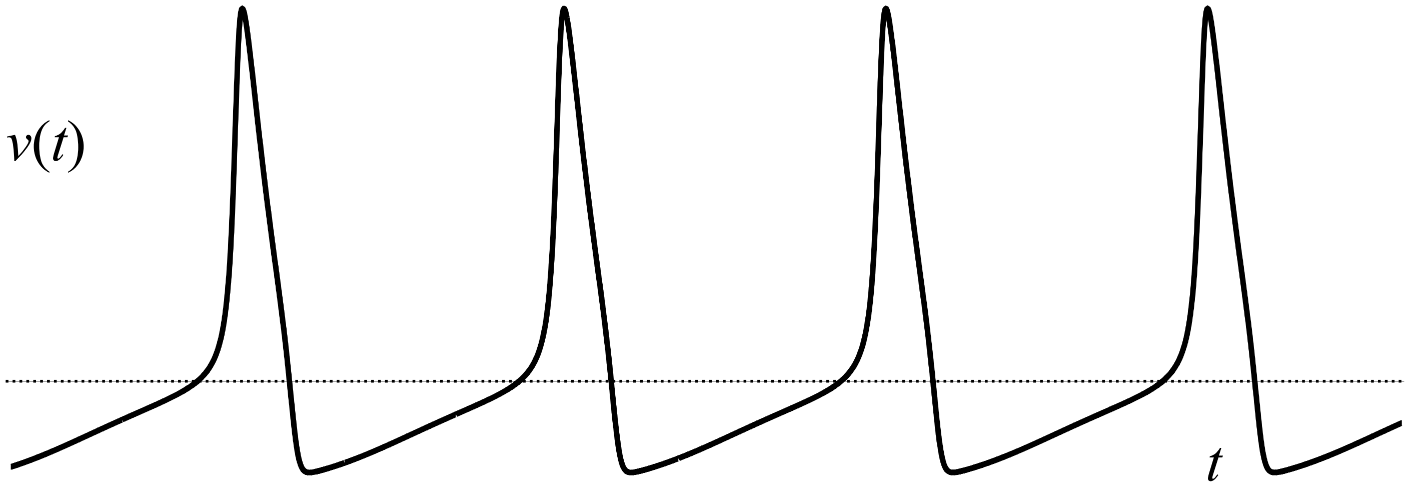
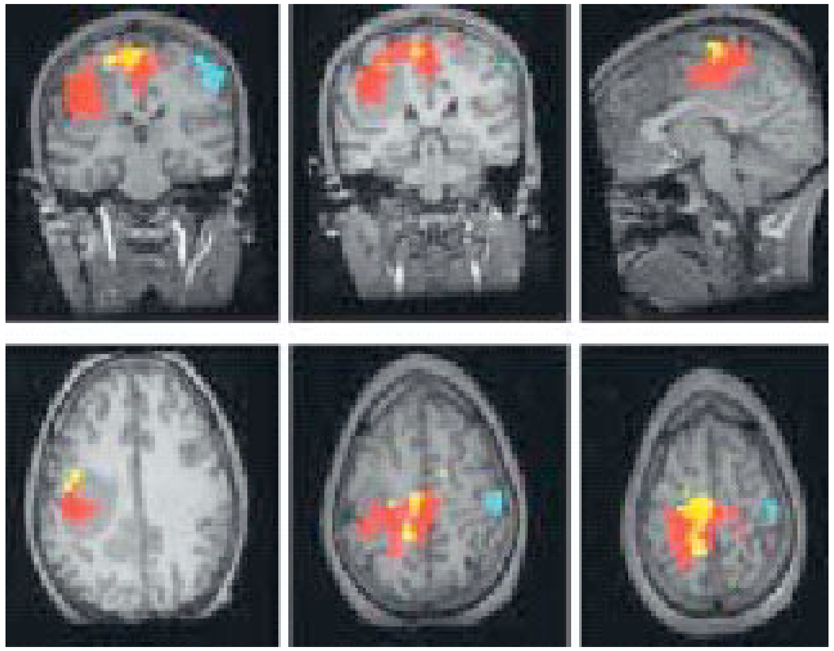
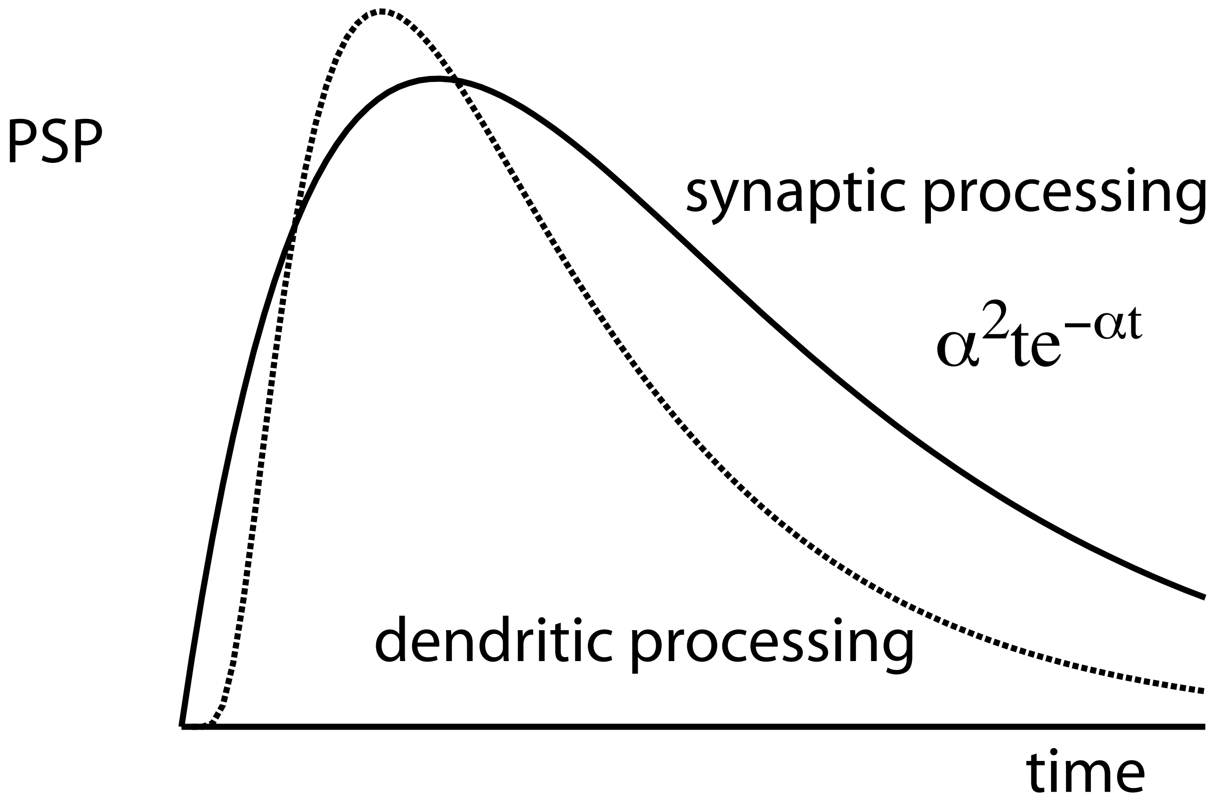




also need a synapse model

$$u(t) = \sum_m \eta(t - T^m)$$



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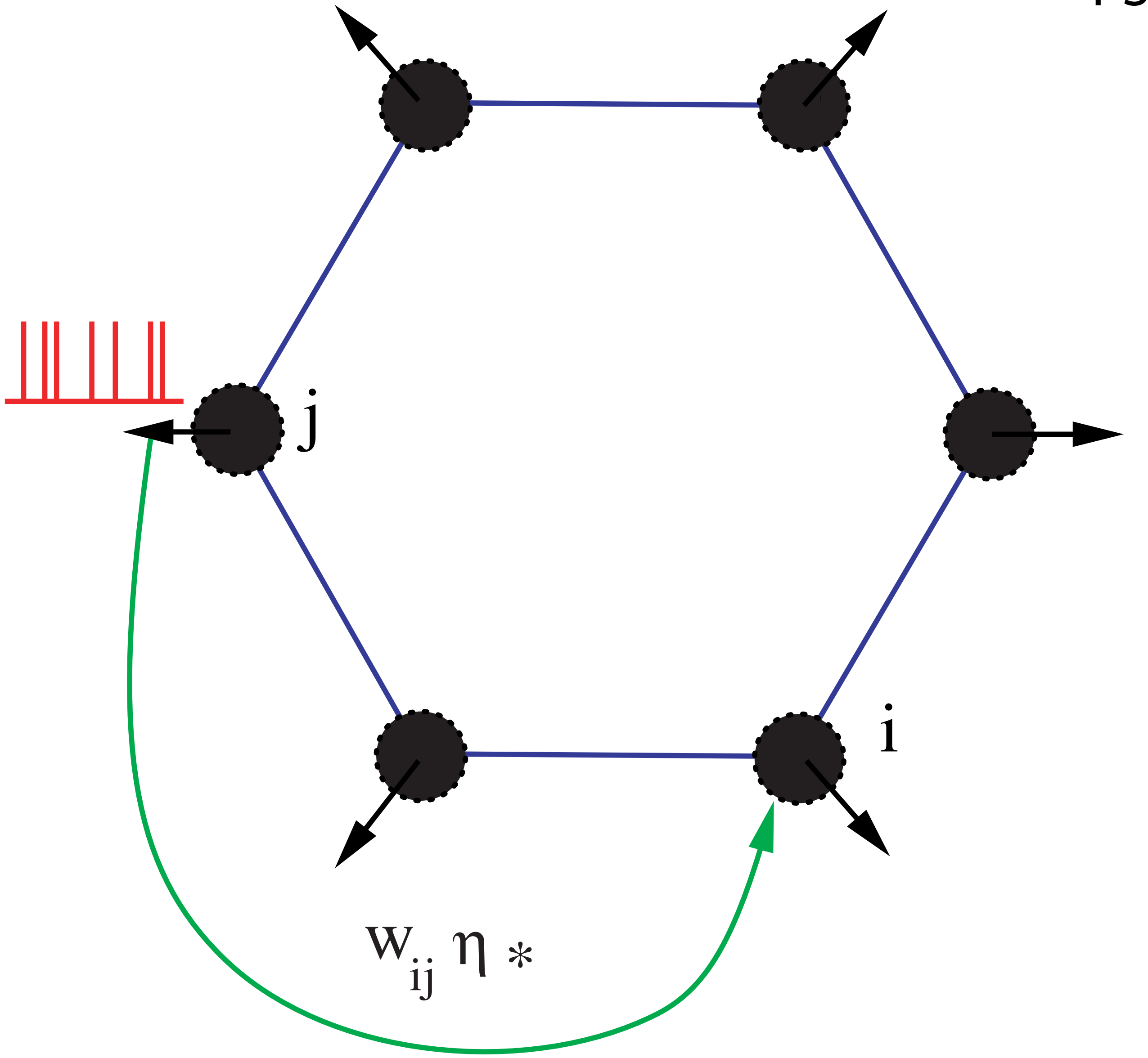

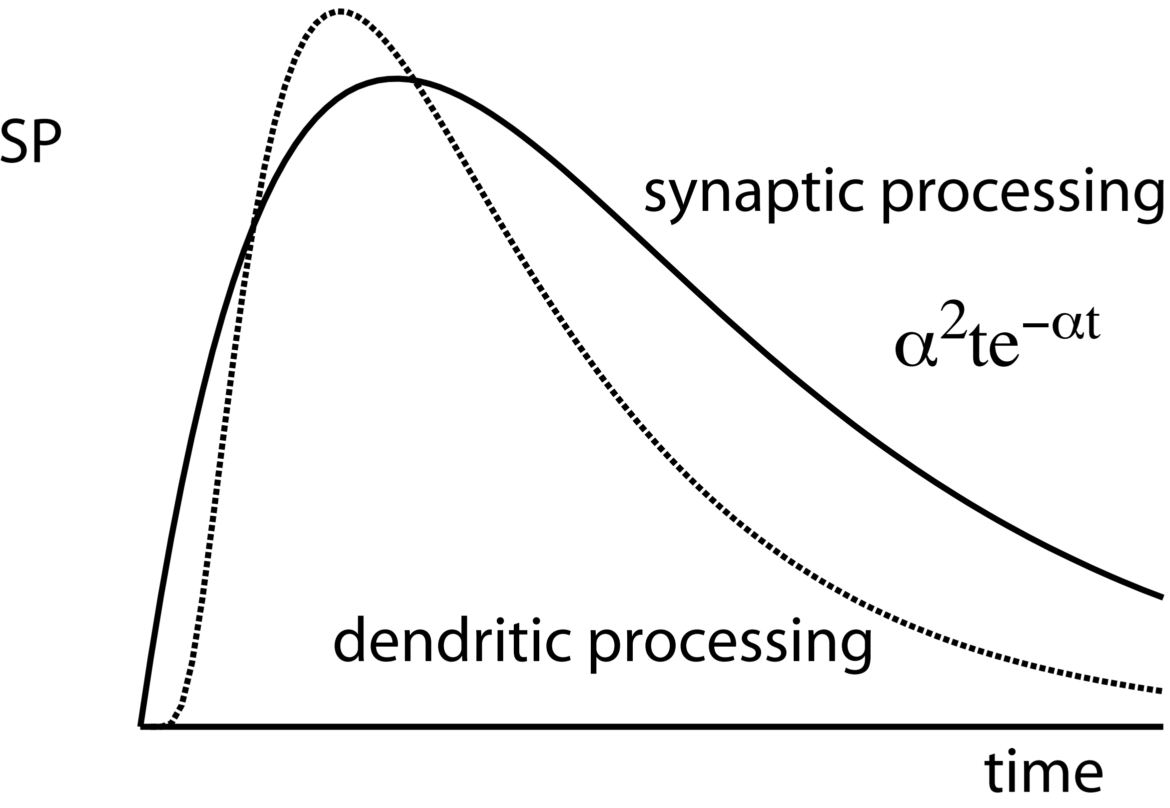

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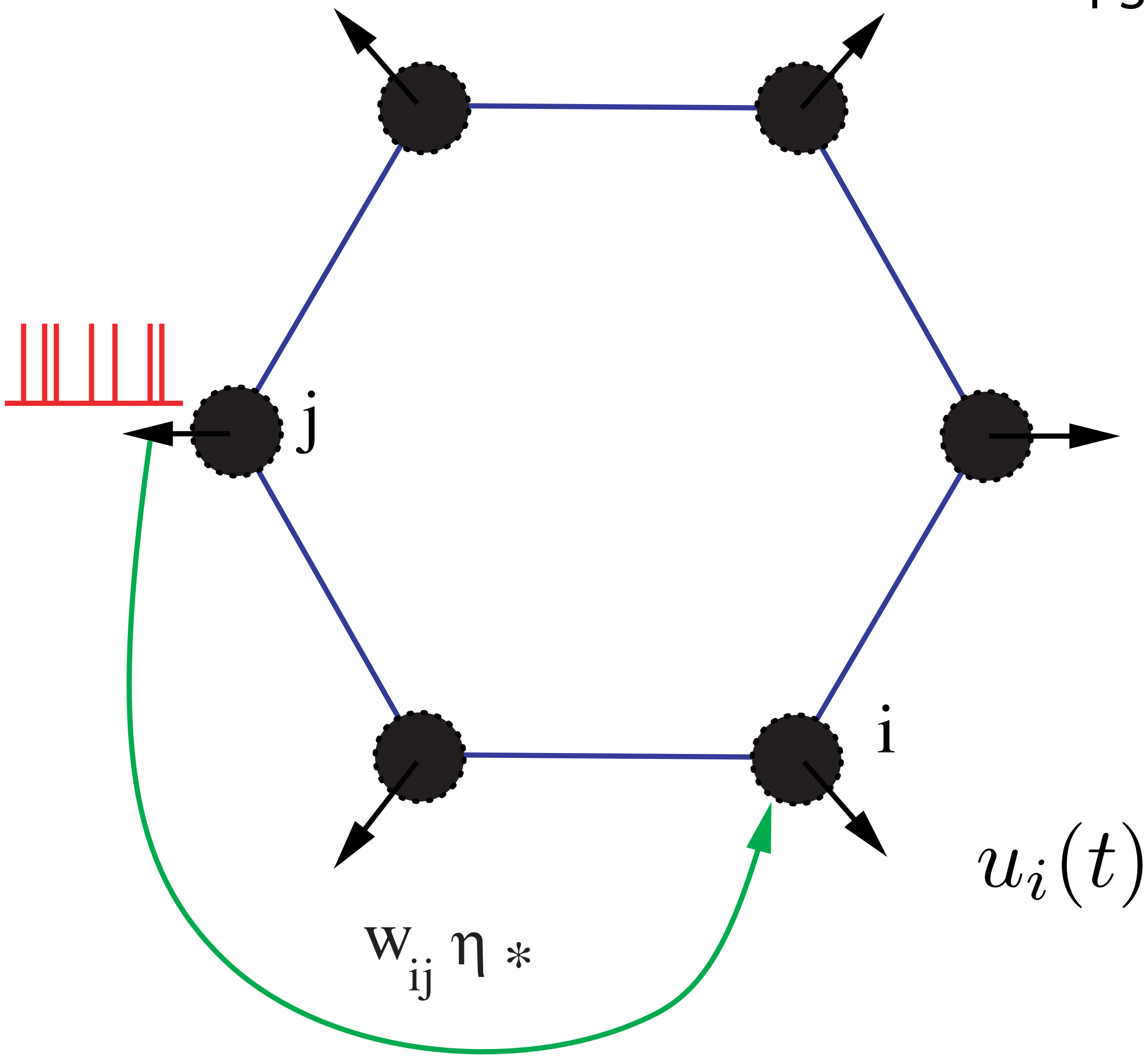

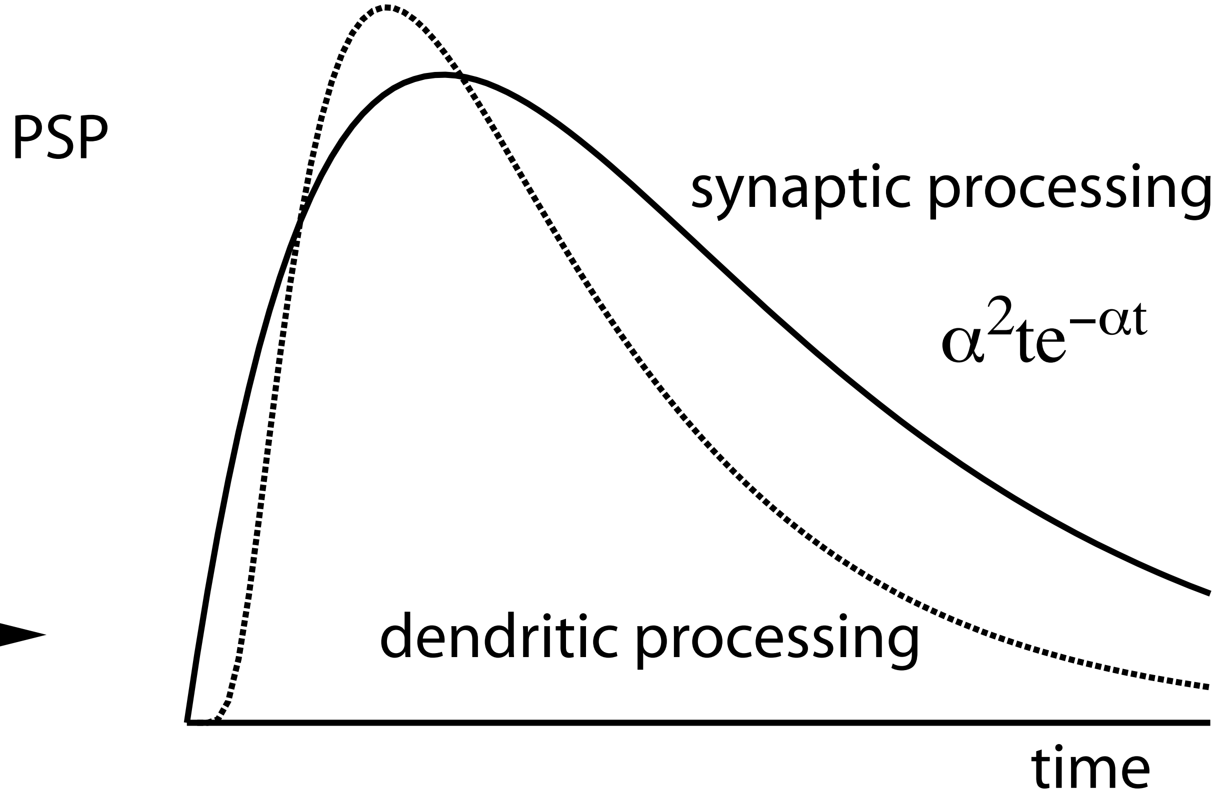

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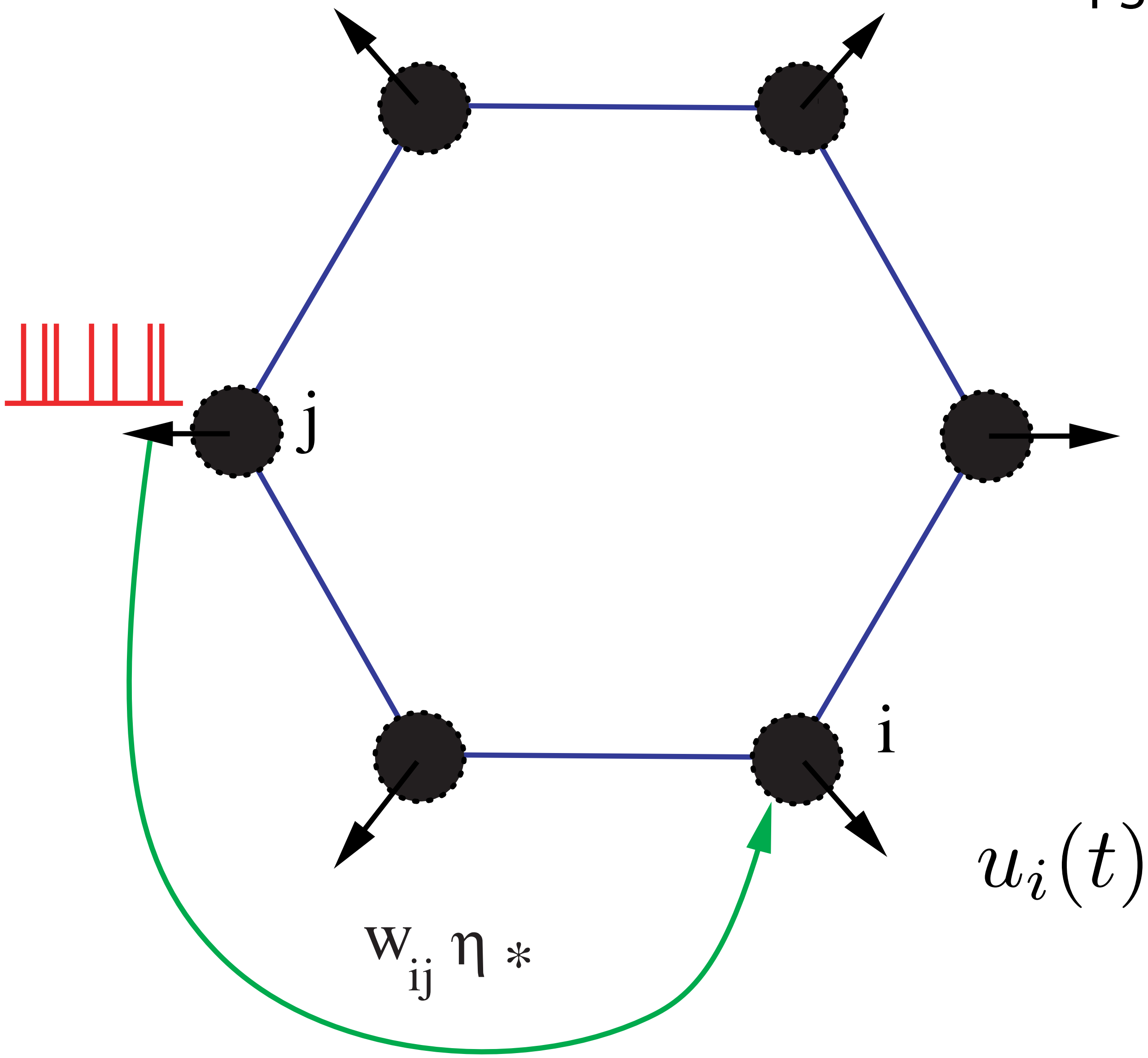

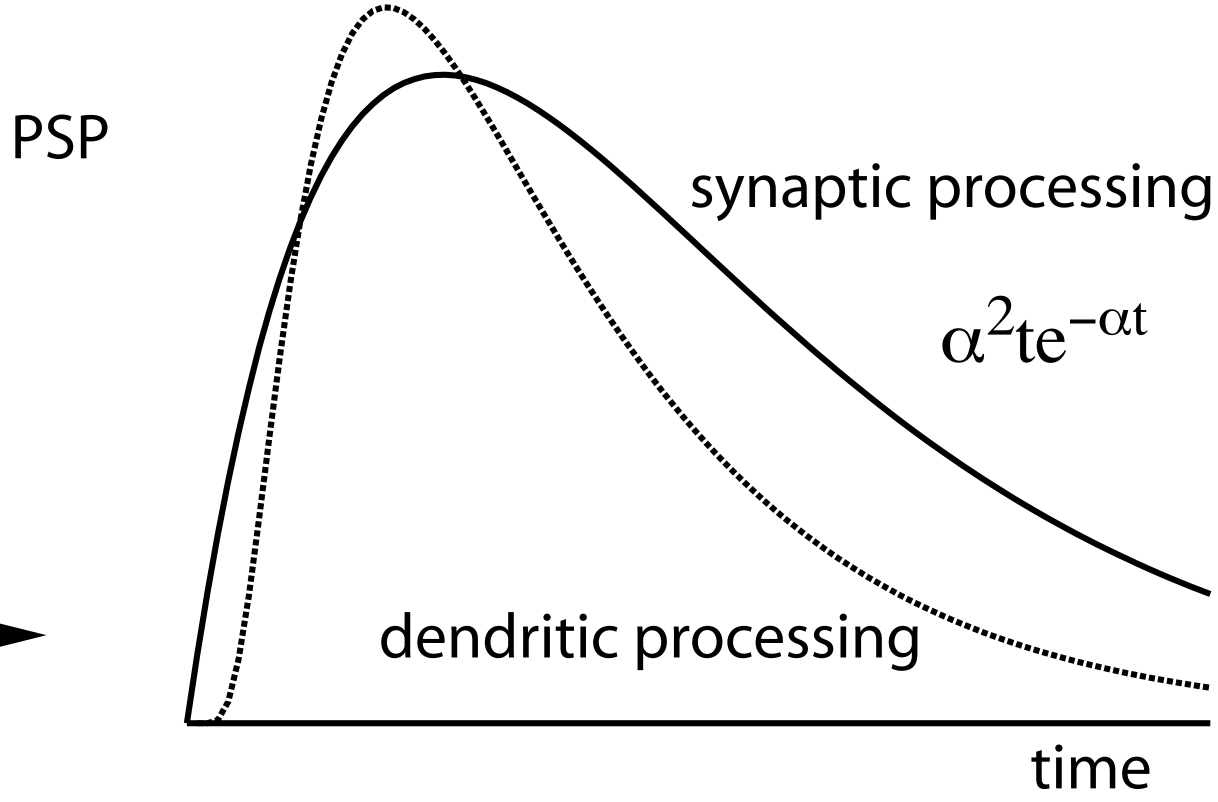

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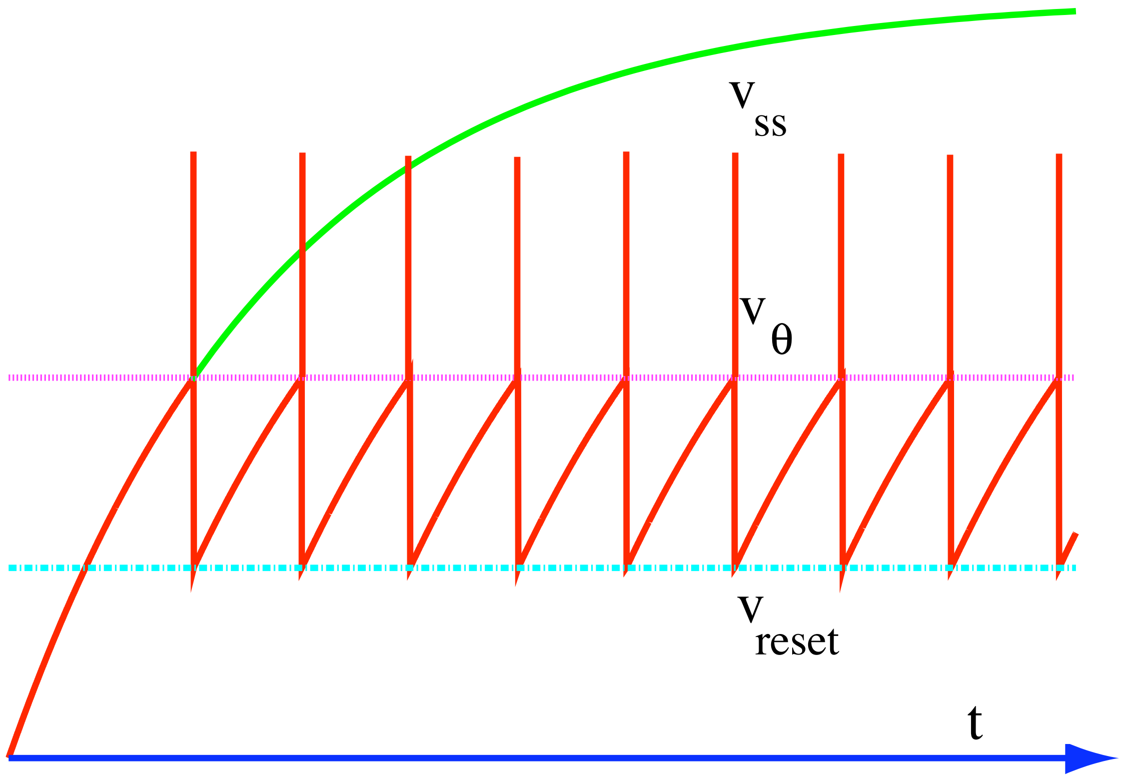
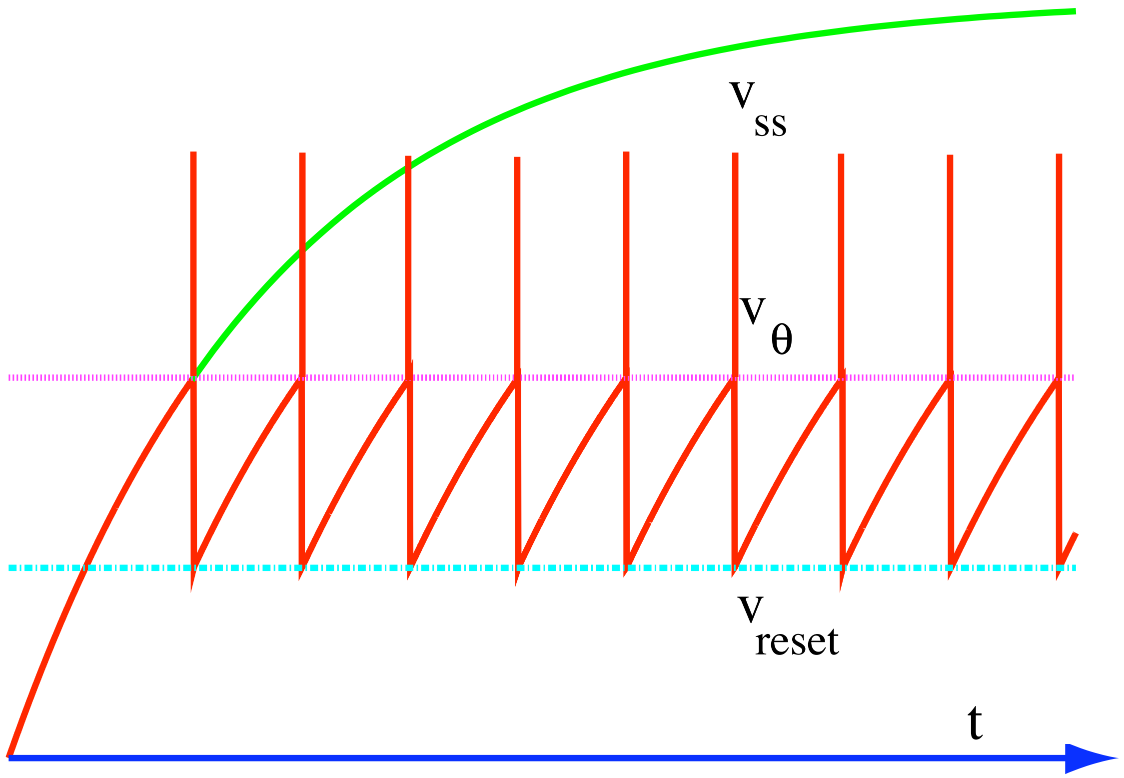
$$= \sum_j w_{ij} \int_0^\infty \eta(s) \sum_m \delta(s - t + T_j^m)\mathrm{ds}$$

Slow synapses: spike train → firing rate f



Firing rate for single neuron model in response to constant current injection is known

Slow synapses: spike train → firing rate *f*



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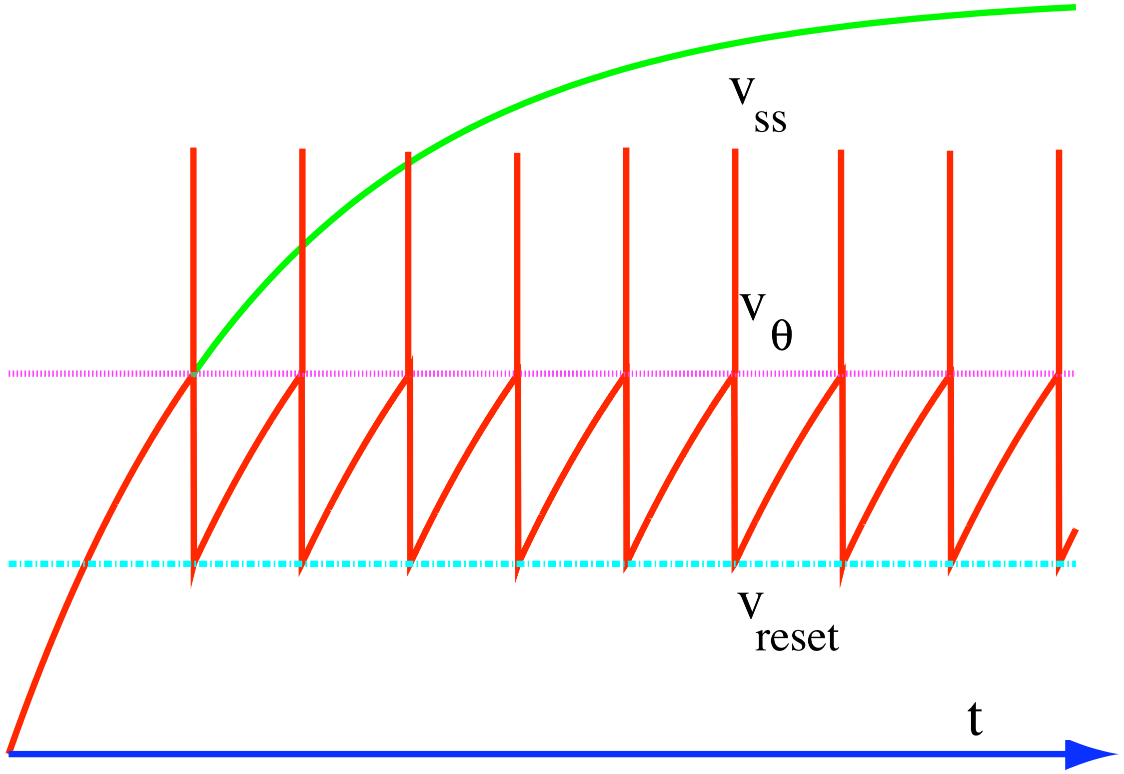

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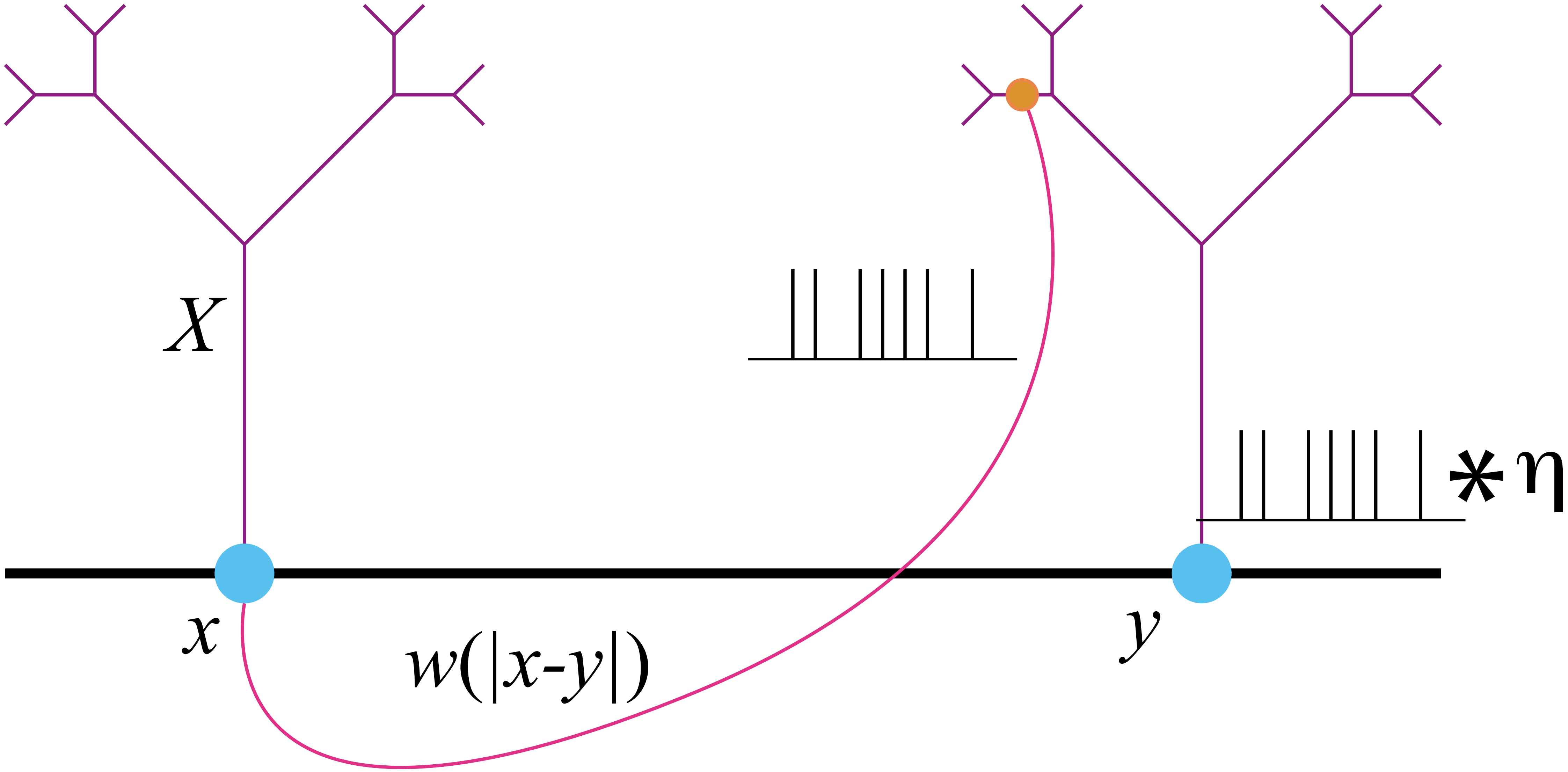
Neural Field Model



Wilson and Cowan (1972, 1973), Amari (1977)

$$u(x,t) = \int_{-\infty}^{\infty} \mathrm{d}y w(y) \int_{0}^{\infty} \mathrm{d}s \eta(s) f(u(x-y, t-s-|y|/v))$$

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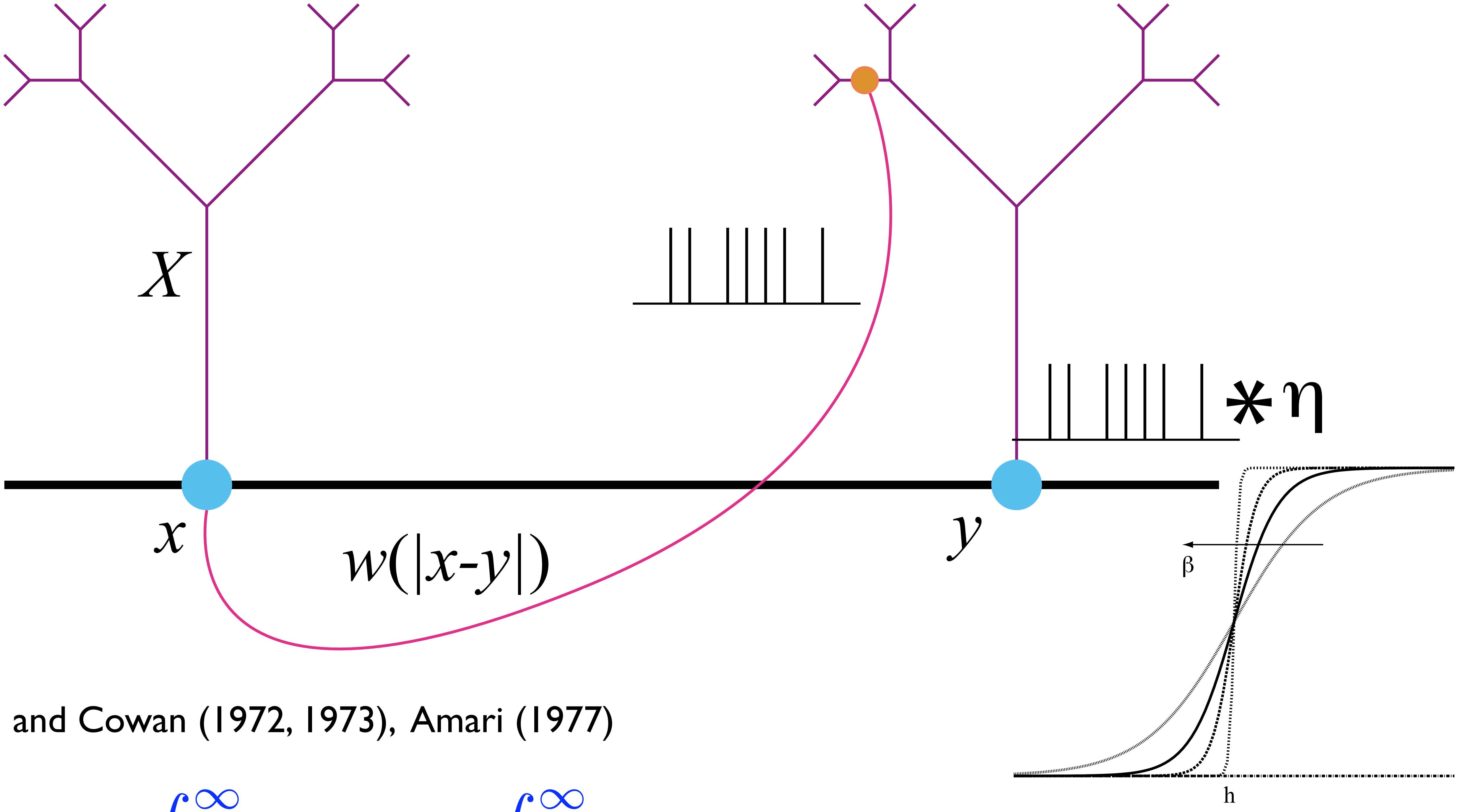

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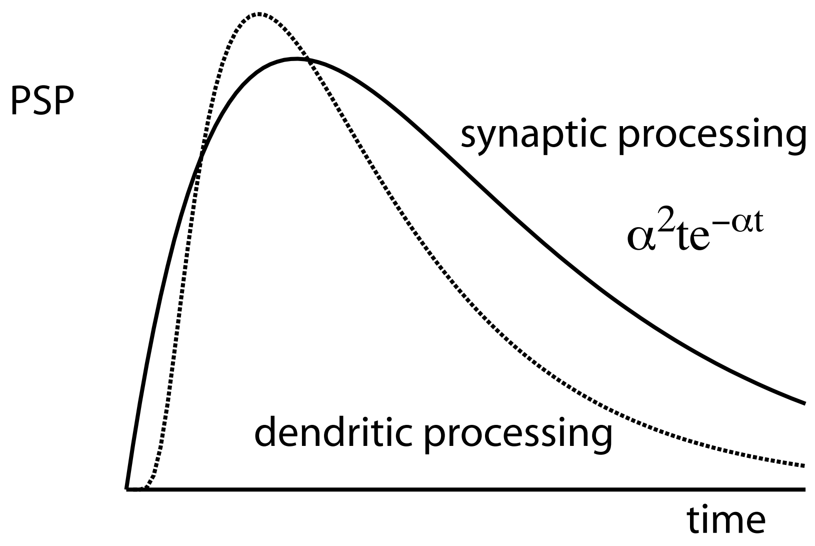
Exponential synapse

$$\eta(t) = \alpha \mathrm{e}^{-\alpha t}, \qquad t > 0$$



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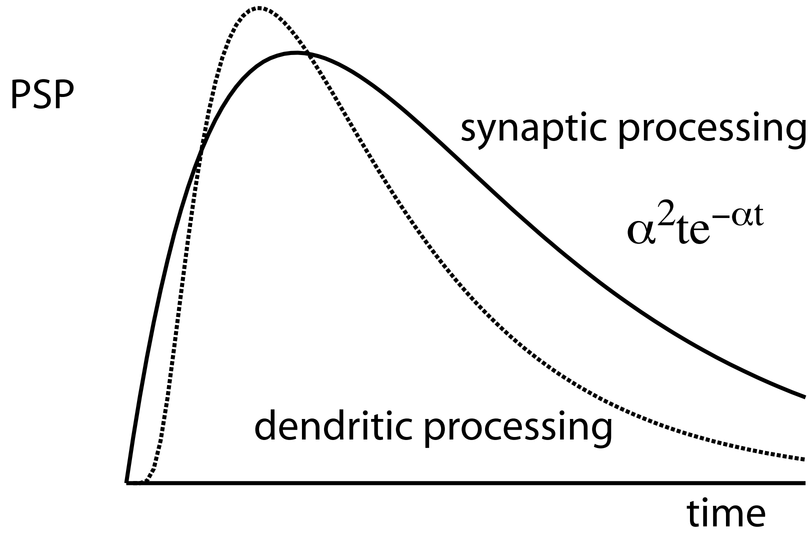

$$\left(1 + \frac{1}{\alpha}\frac{\partial}{\partial t}\right) u = \psi$$

$$\psi(x,t) = \int_{-\infty}^{\infty} w(x-y) f(u(y, t - |x-y|/v)) \mathrm{d}y$$

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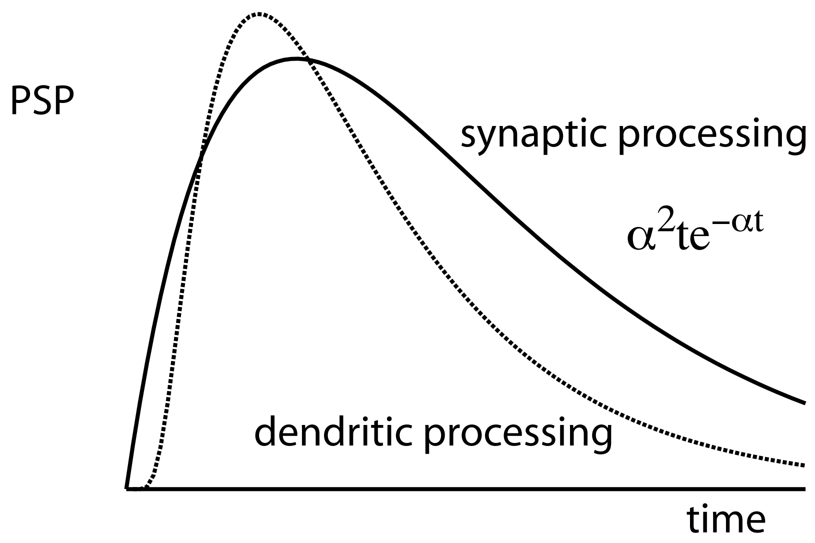

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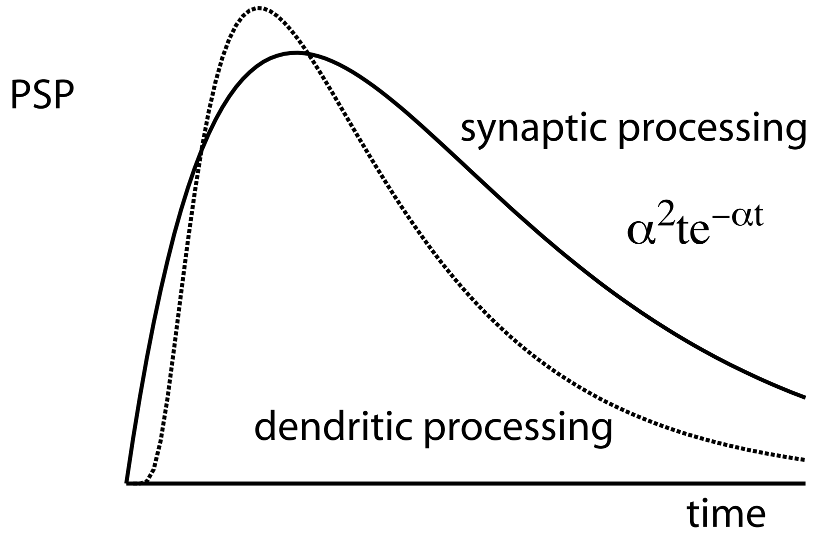

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exploit
convolution
structure

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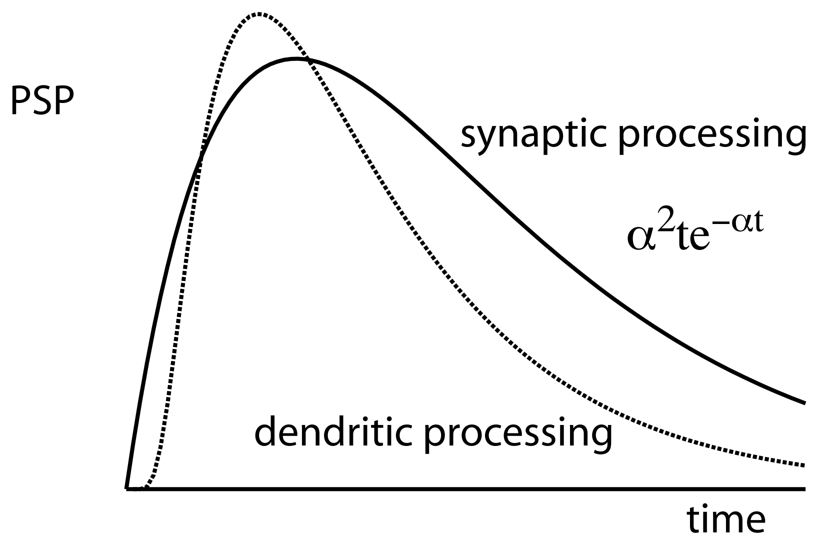

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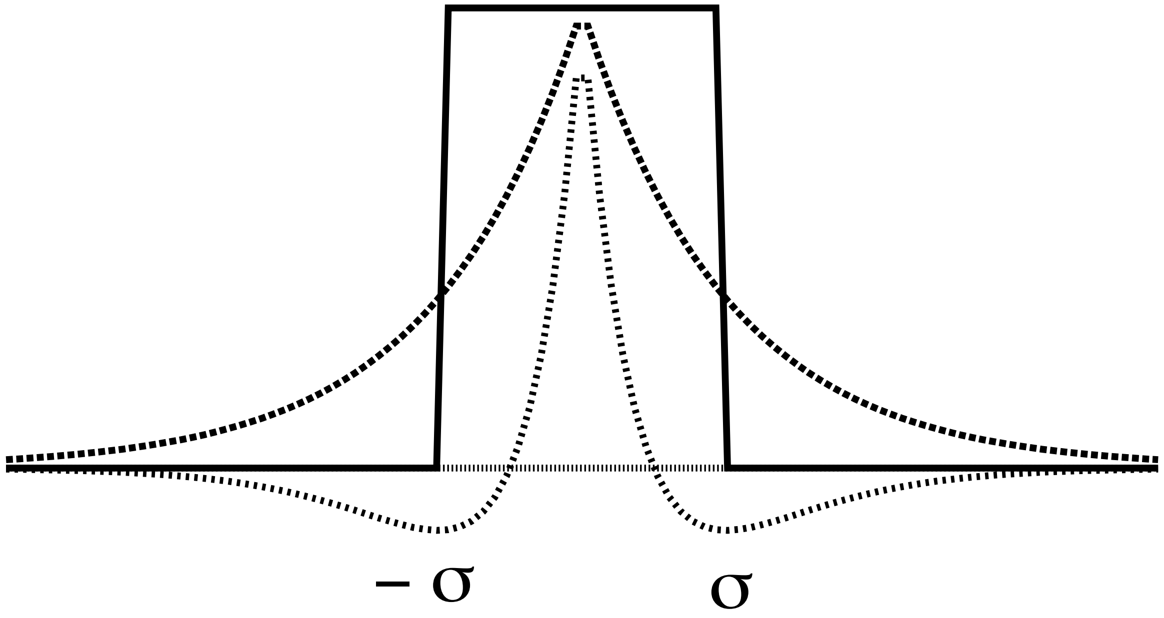
$$\psi(k,\omega) = G(k,\omega)\rho(k,\omega)$$

Exponential anatomy

$$w(x) = \mathrm{e}^{-|x|}/2$$



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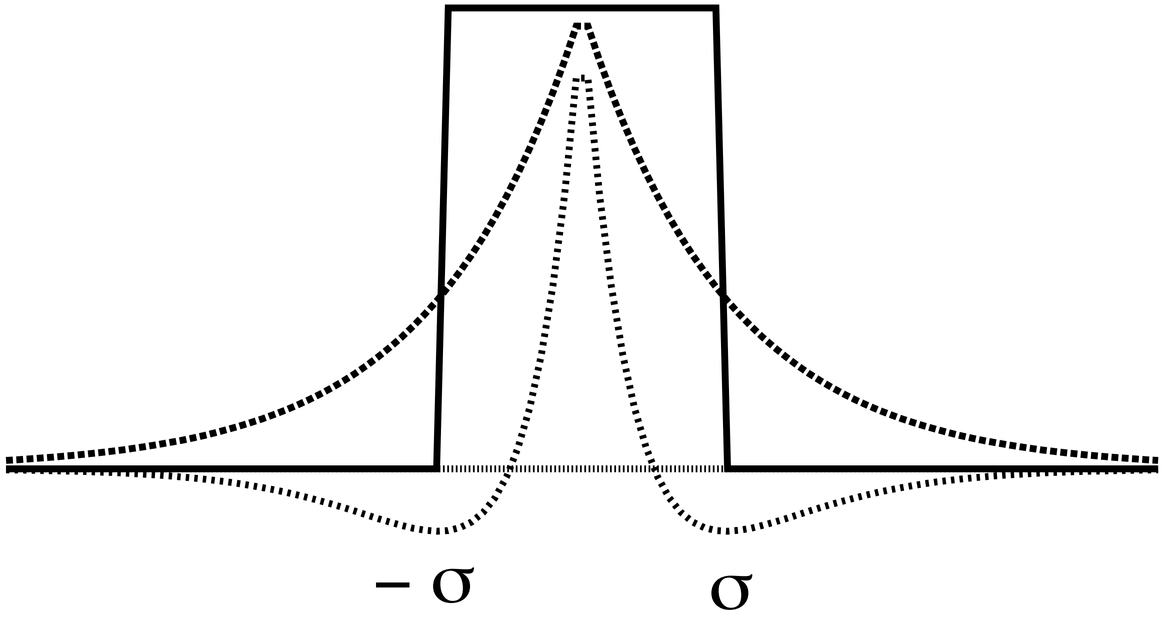

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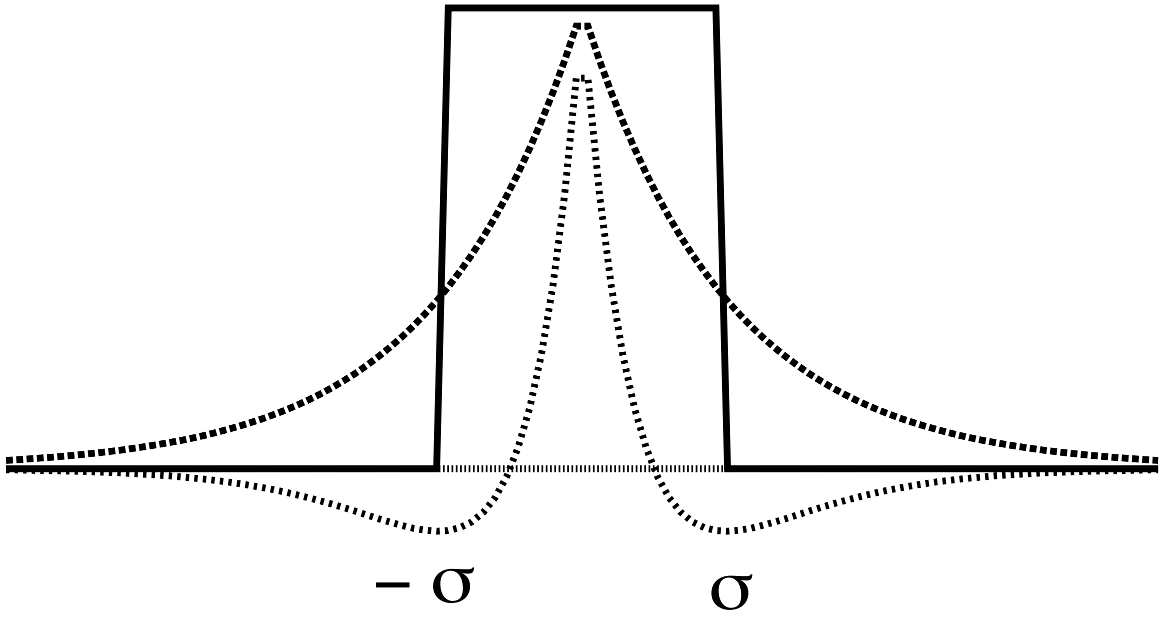

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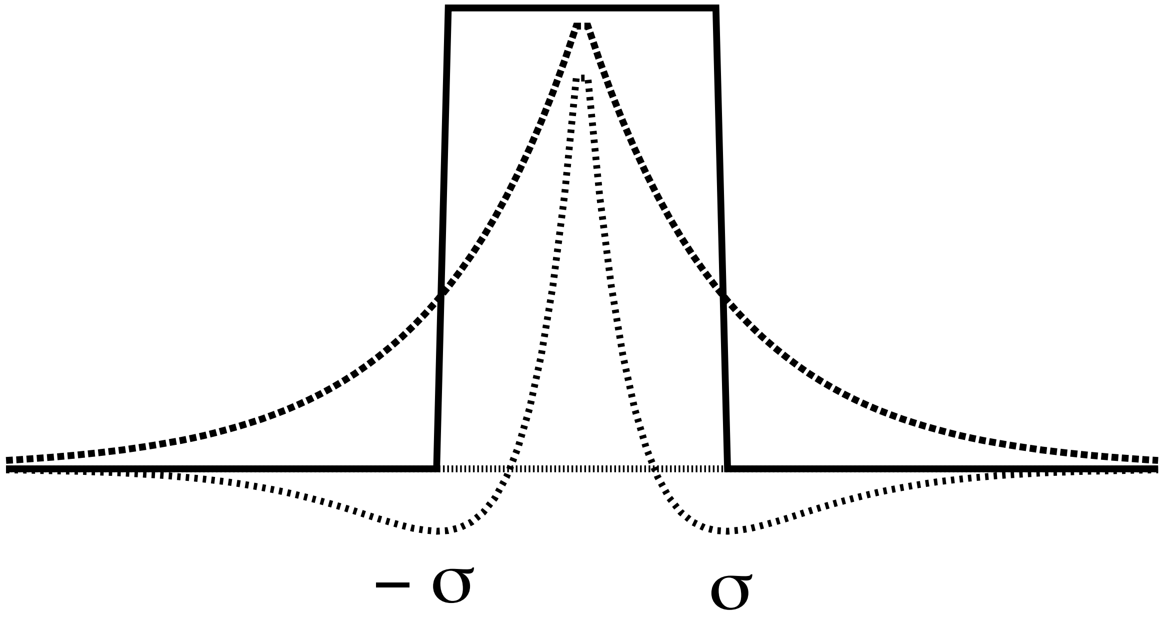


$$\left[\left(1 + i\frac{\omega}{v}\right)^2 + k^2\right]\psi(k,\omega) = \left[1 + i\frac{\omega}{v}\right]\rho(k,\omega)$$

$$k^2 \leftrightarrow -\nabla^2 \text{ and } i\omega \leftrightarrow \partial_t$$

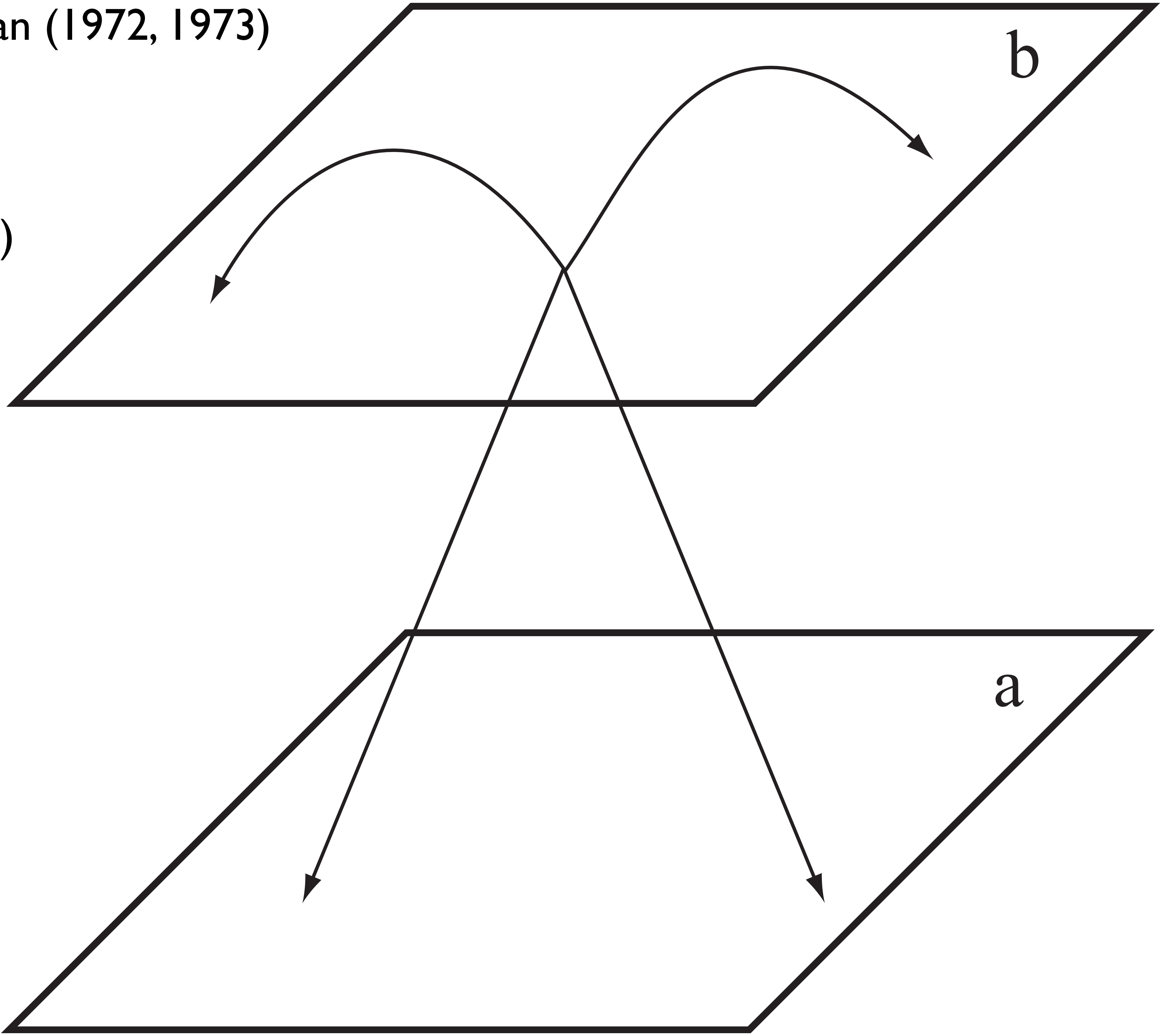
PDE in terms of the local operators $\nabla^2, \quad \partial_t$

$$[(v + \partial_t)^2 - v^2\partial_{xx}]\psi(x,t) = [v^2 + v\partial_t] f \circ u(x,t)$$

Damped inhomogeneous brain-wave equation

2D Brain-wave equation

Wilson and Cowan (1972, 1973)
Nunez (1974)
Amari (1977)
Liley, Cadusch
and Wright (1999)



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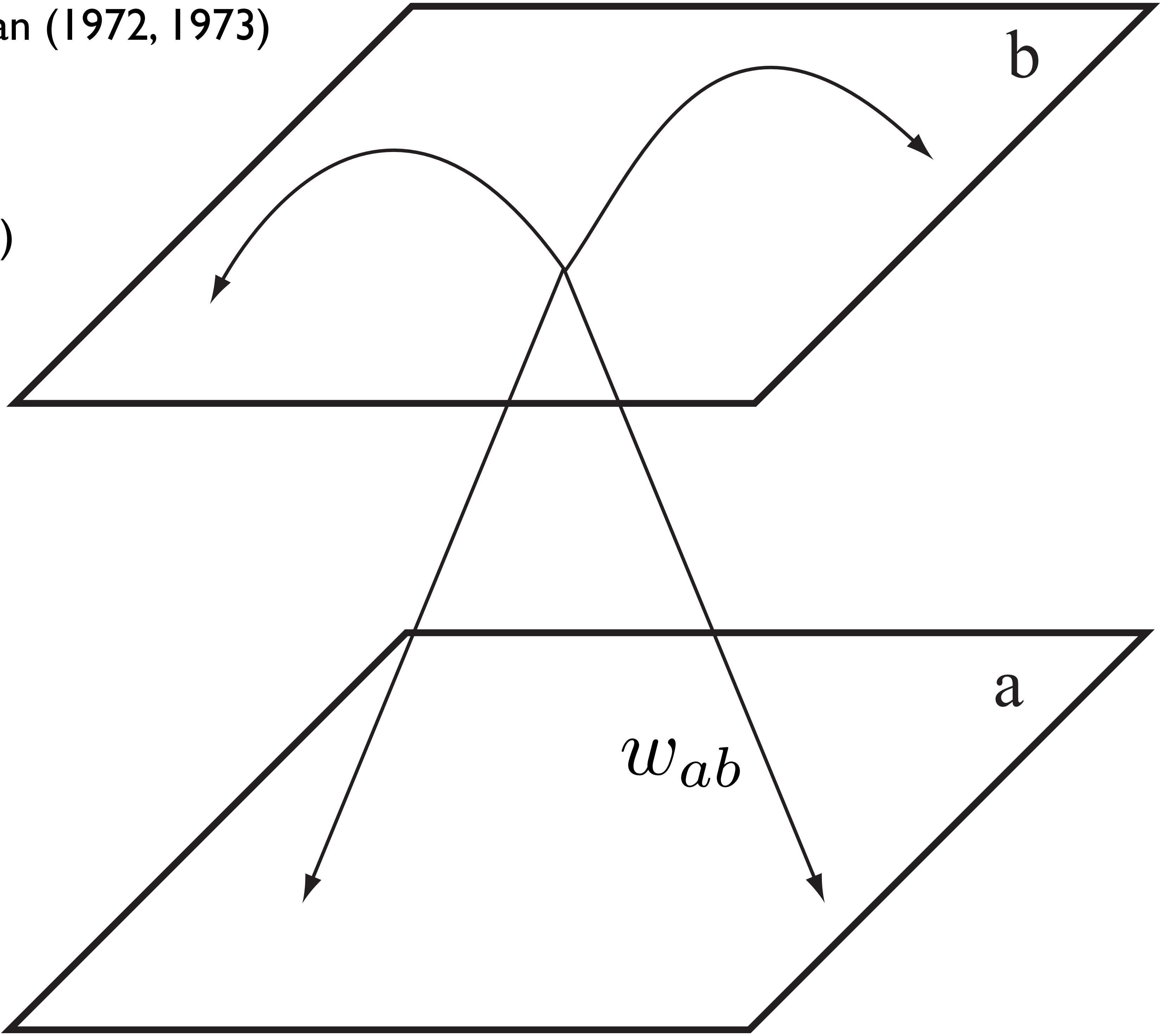

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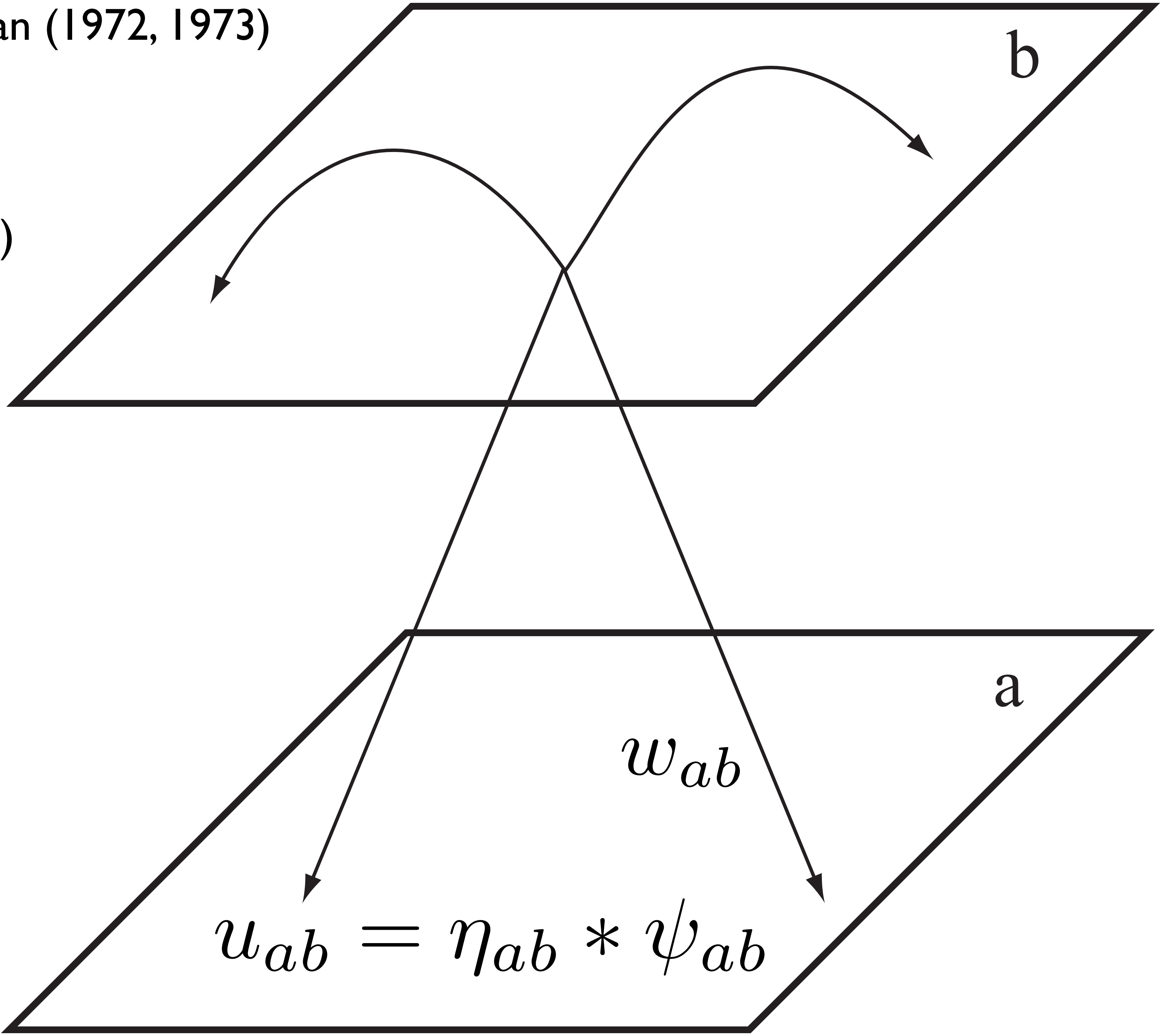

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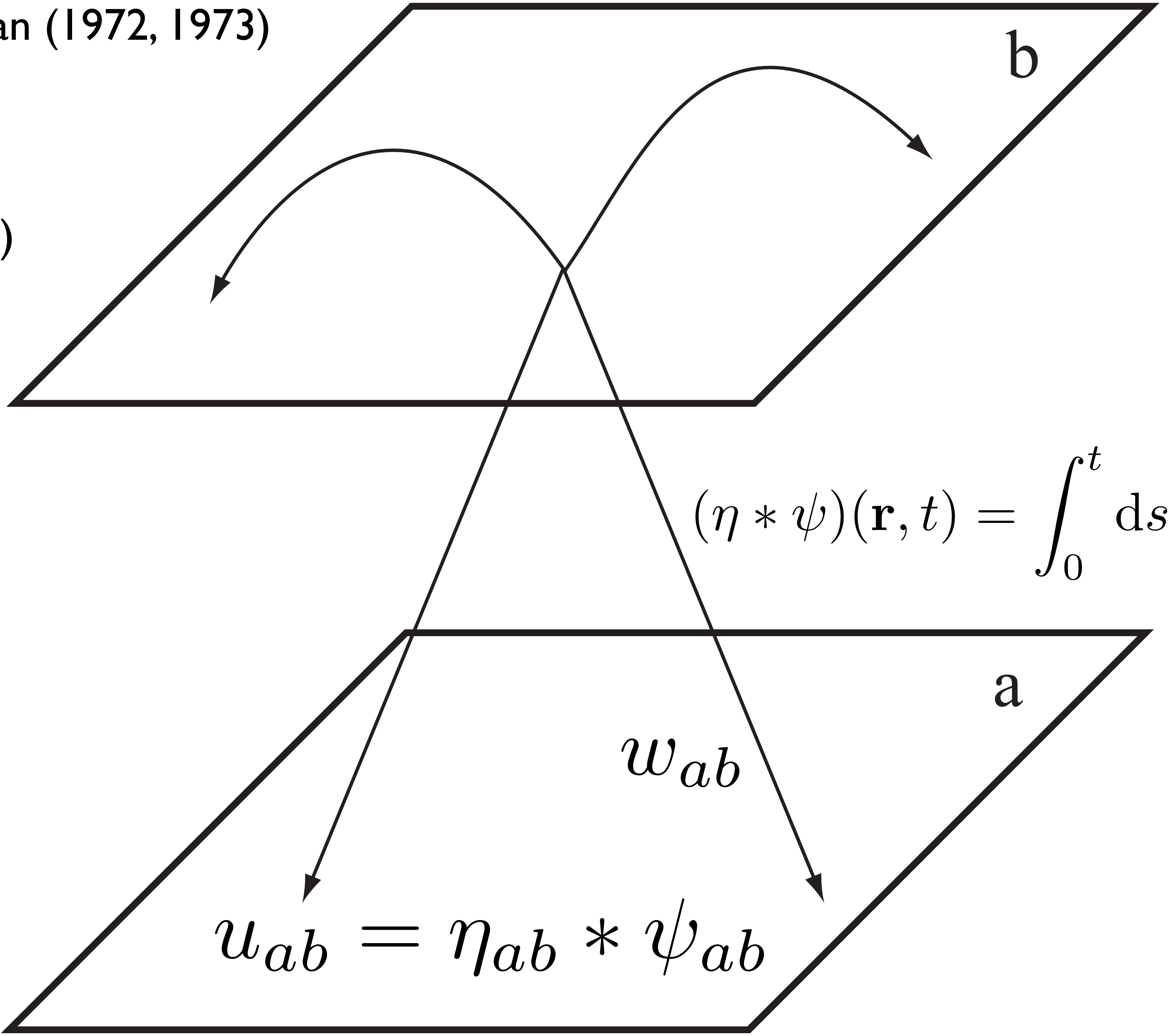

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$$u_{ab} = \eta_{ab} * \psi_{ab}$$

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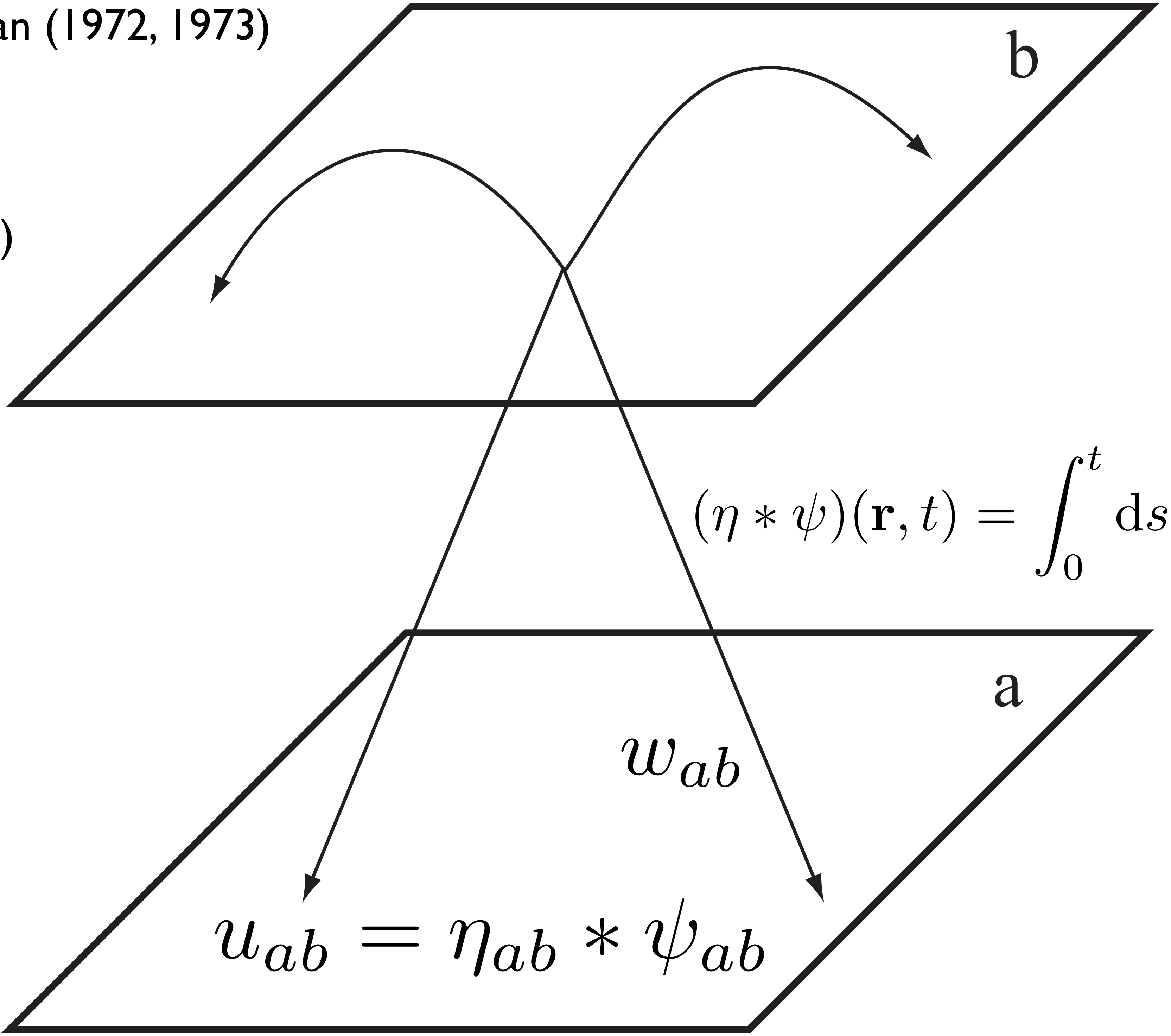

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Liley, Cadusch
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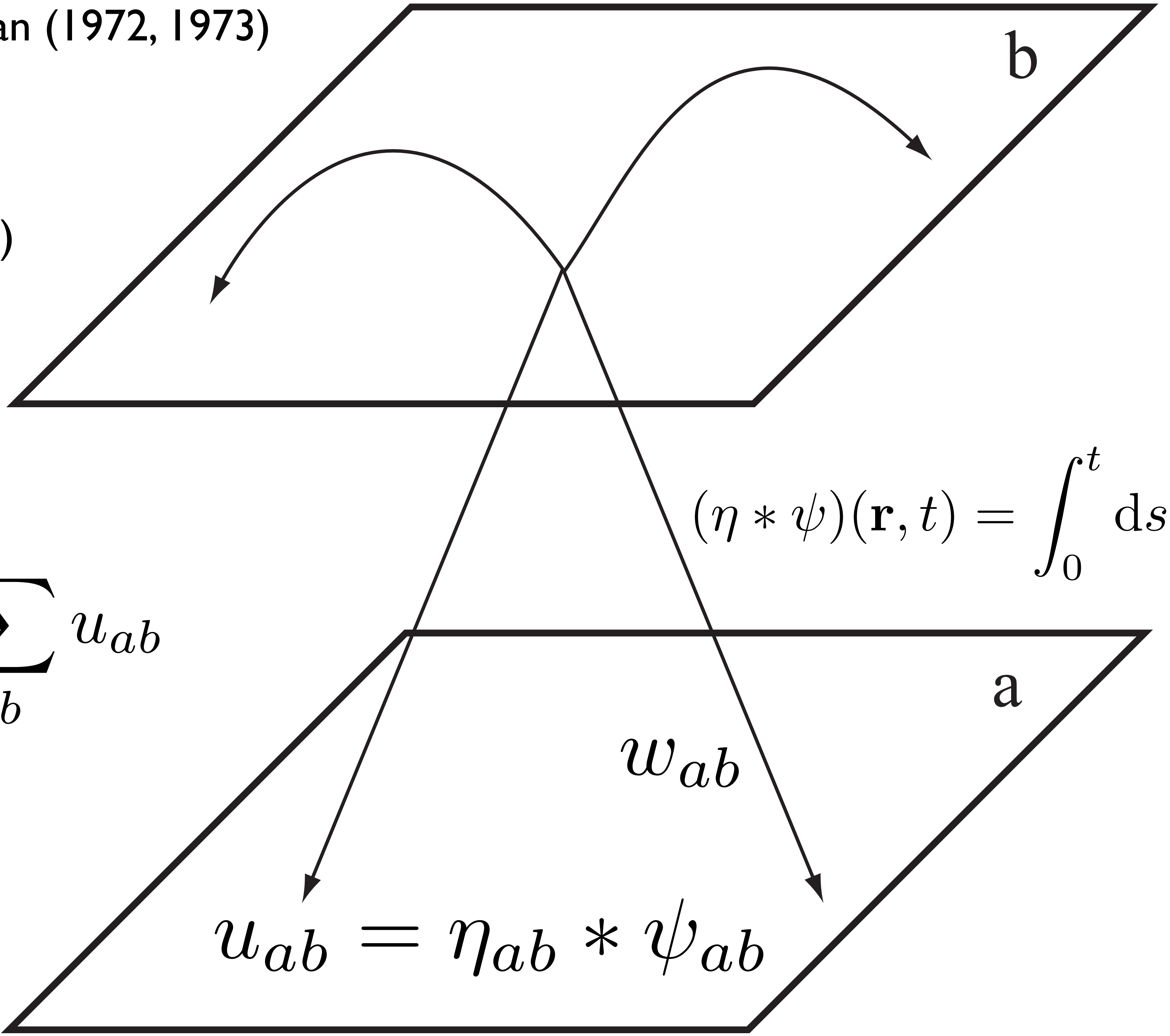

$$u_{ab} = \eta_{ab} * \psi_{ab}$$

$$(\eta * \psi)(\mathbf{r}, t) = \int_0^t \mathrm{d}s \eta(s) \psi(\mathbf{r}, t - s)$$

$$h_a = \sum_b u_{ab}$$

$$\psi_{ab}(\mathbf{r}, t) = \int_{\mathbb{R}^2} \mathrm{d}\mathbf{r}' w_{ab}(\mathbf{r}, \mathbf{r}') f_b \circ h_b(\mathbf{r}', t - |\mathbf{r} - \mathbf{r}'| / v_{ab})$$

No such luck for the natural choice

$$w_{ab}(r) = w_{ab}^{0} \mathrm{e}^{-r/\sigma_{ab}}/(2\pi)$$

$$G_{ab}(k,\omega) = w_{ab}^{0} \frac{A_{ab}(\omega)}{(A_{ab}^{2}(\omega) + k^{2})^{3/2}}$$

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Introducing the operator $\mathcal{A}_{ab}$:

$$\mathcal{A}_{ab} = \left(\frac{1}{\sigma_{ab}} + \frac{1}{v_{ab}}\partial_t\right)^2$$

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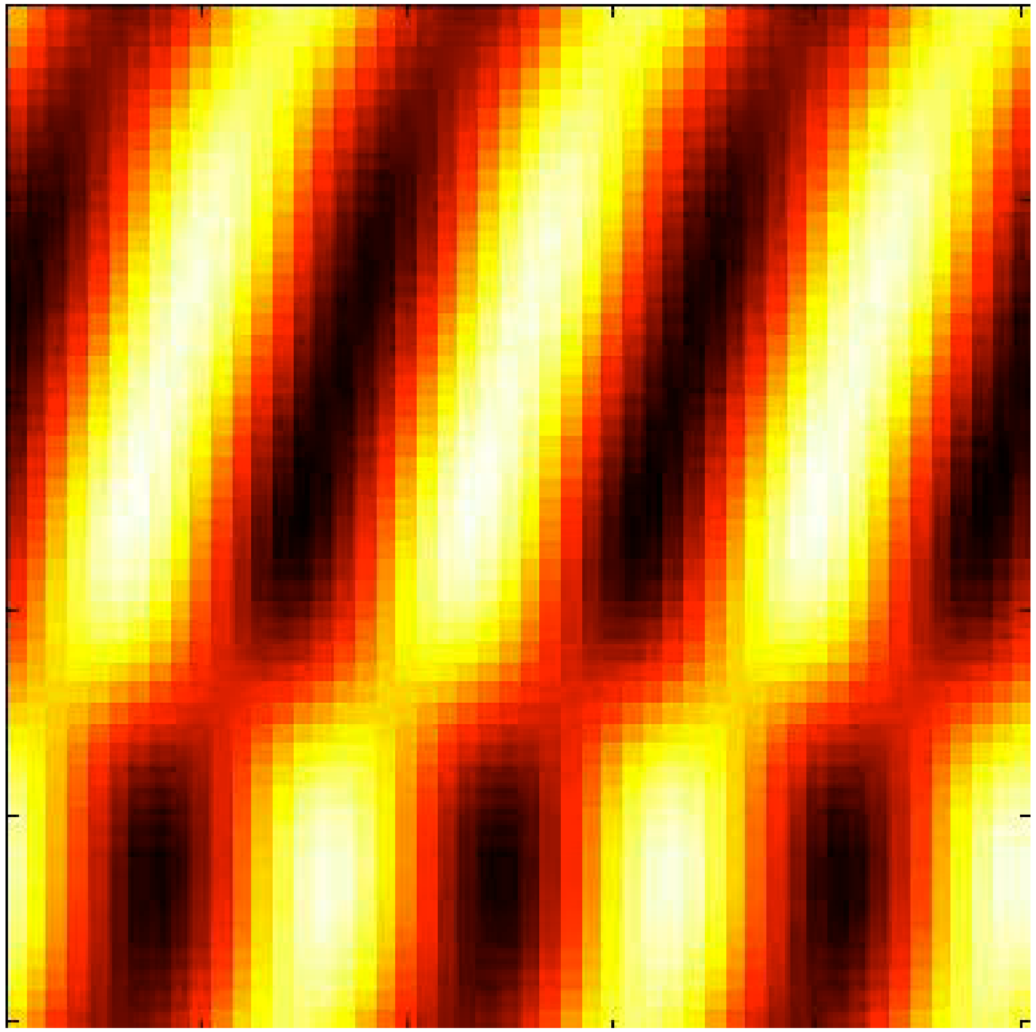
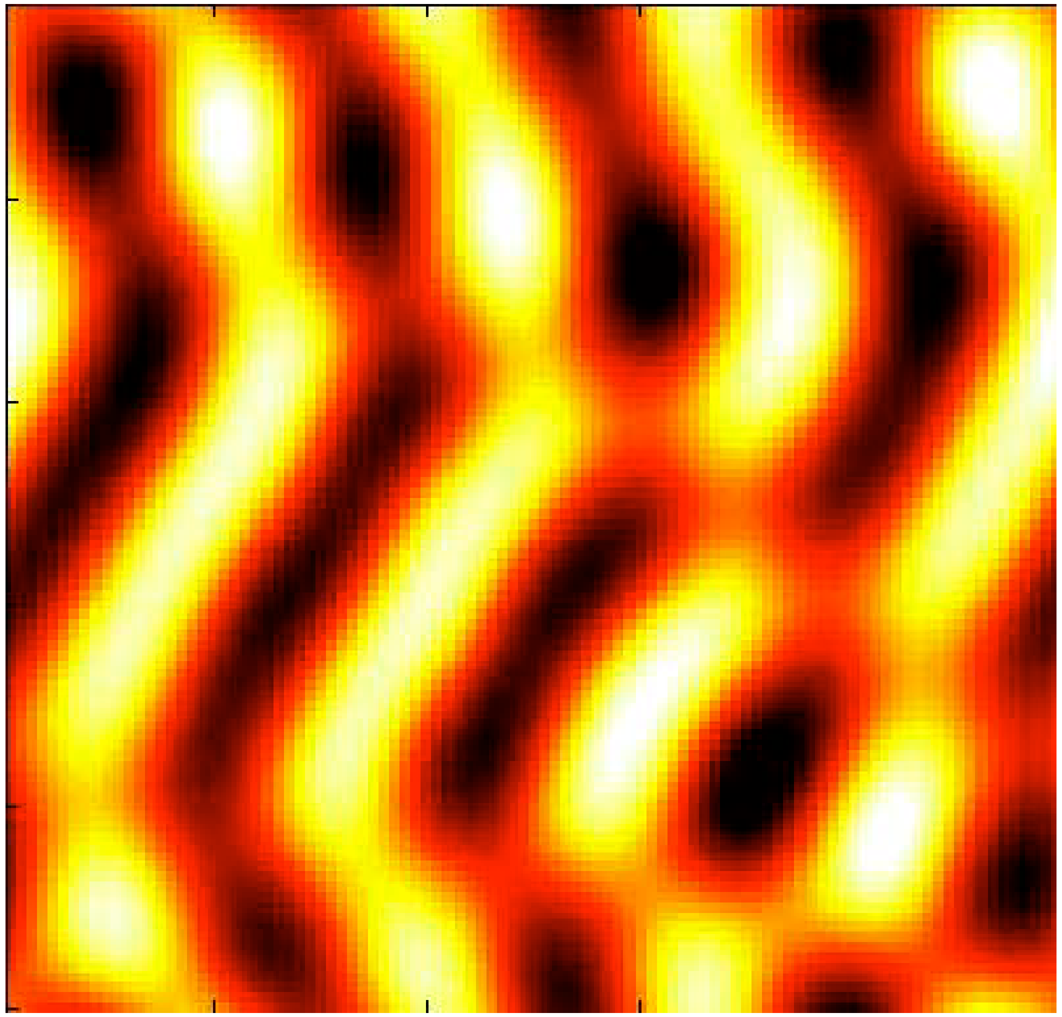
Better *rational* approximation

S Coombes, N A Venkov, L Shiau, I Bojak, D T J Liley and C R Laing 2007 Modeling electrocortical activity through improved local approximations of integral neural field equations, Physical Review E, Vol 76, 051901-8. (Open access).

Eye candy (for a two-layer E-I network)



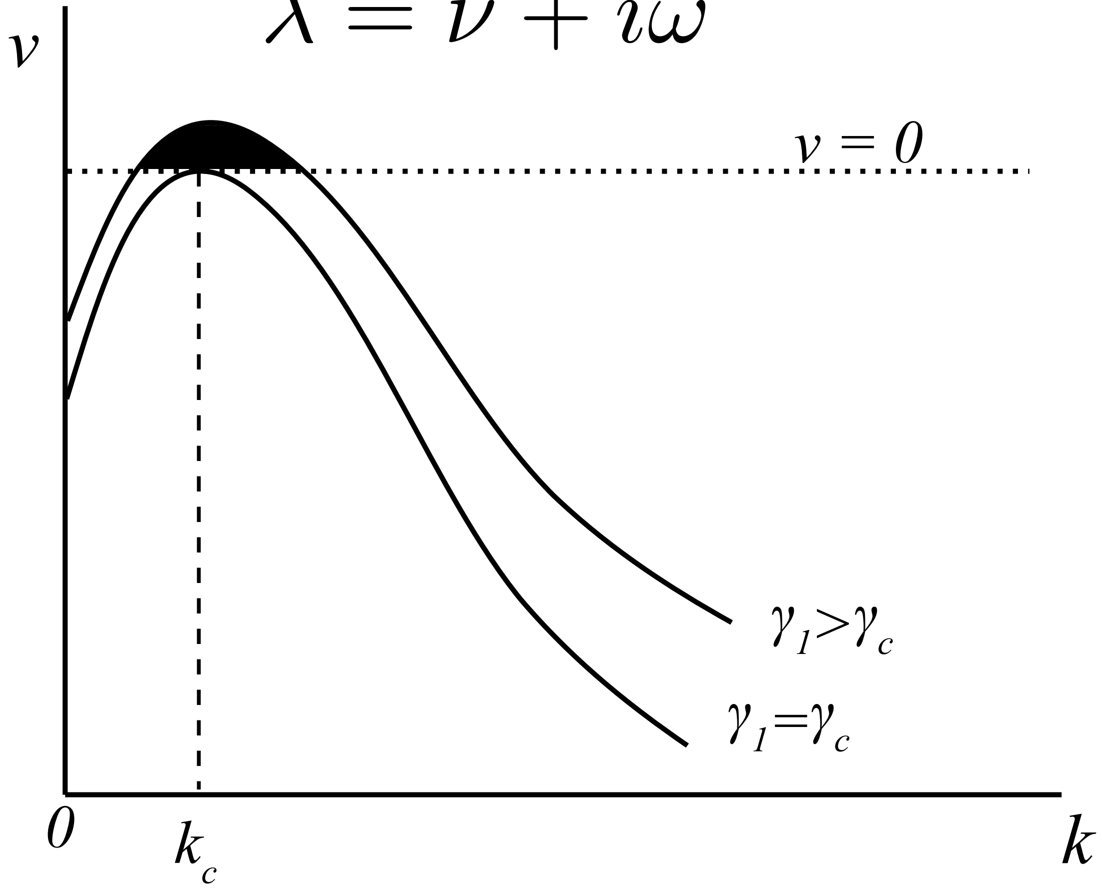
Long-wavelength

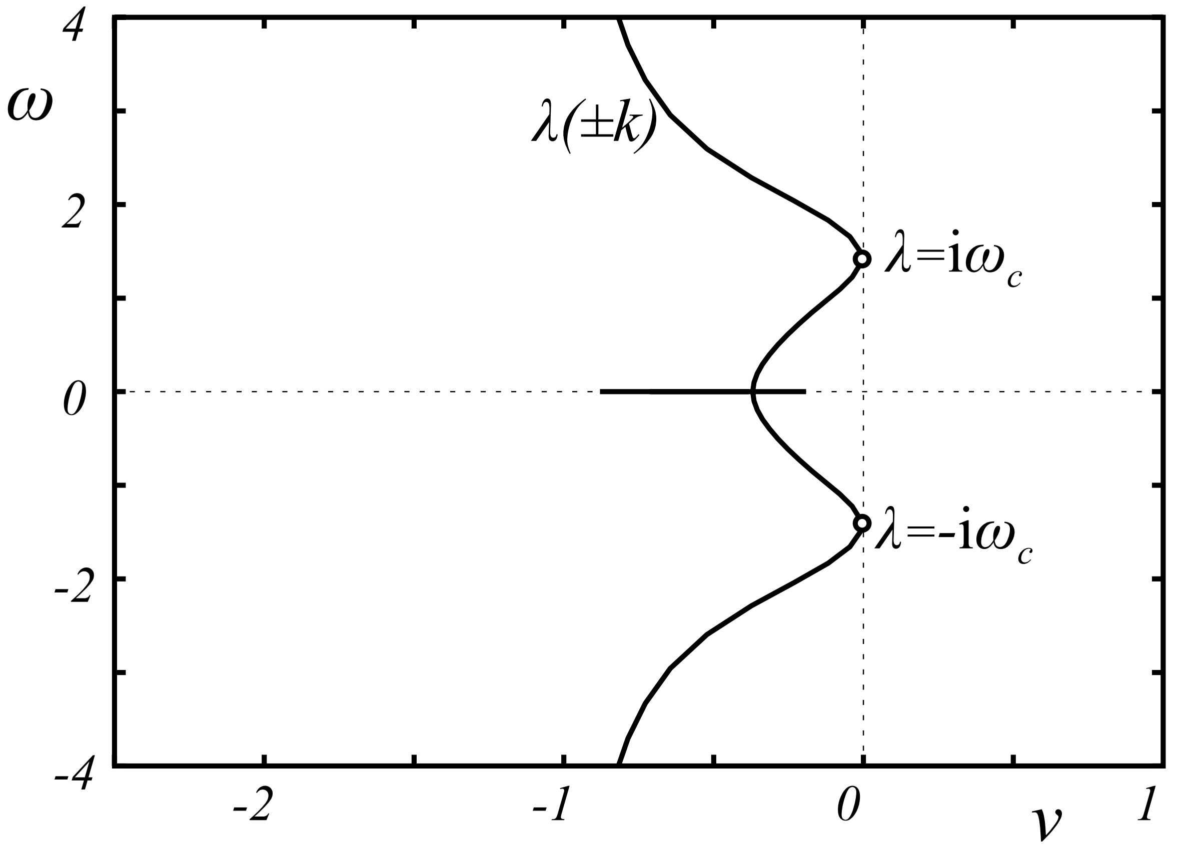


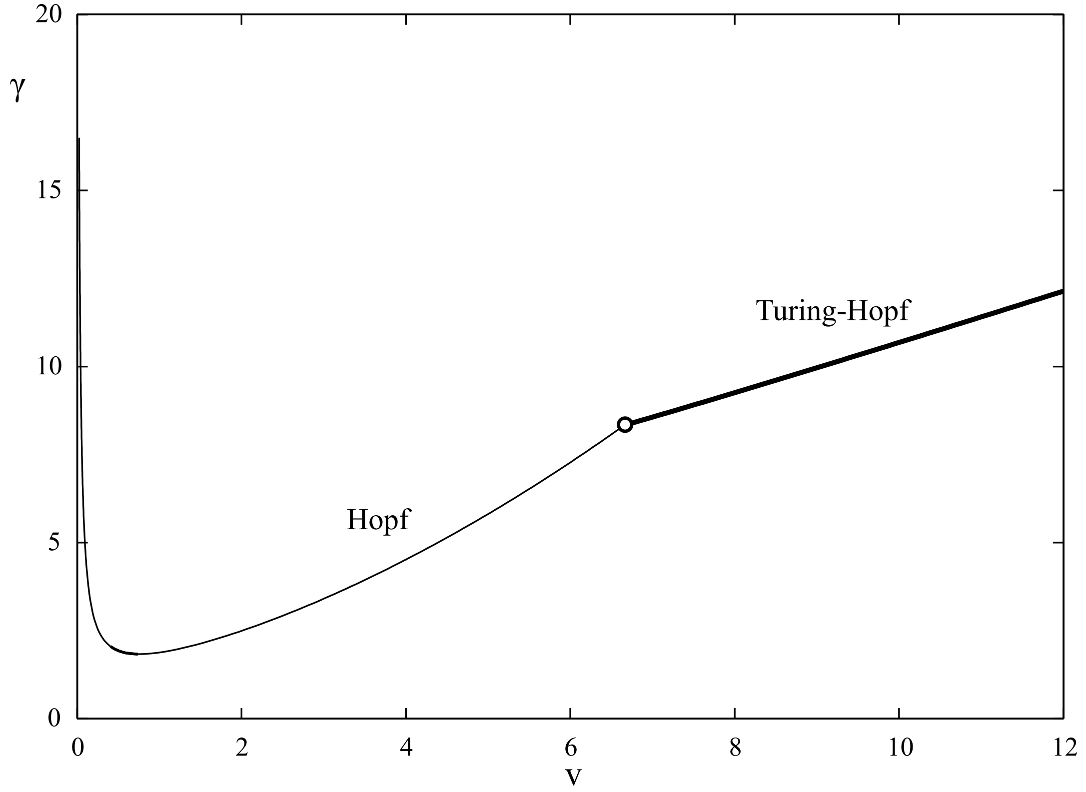
Rational

Dynamic Turing instability $\mathrm{e}^{\lambda t}\mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}}$

$$\lambda = \nu + i\omega$$







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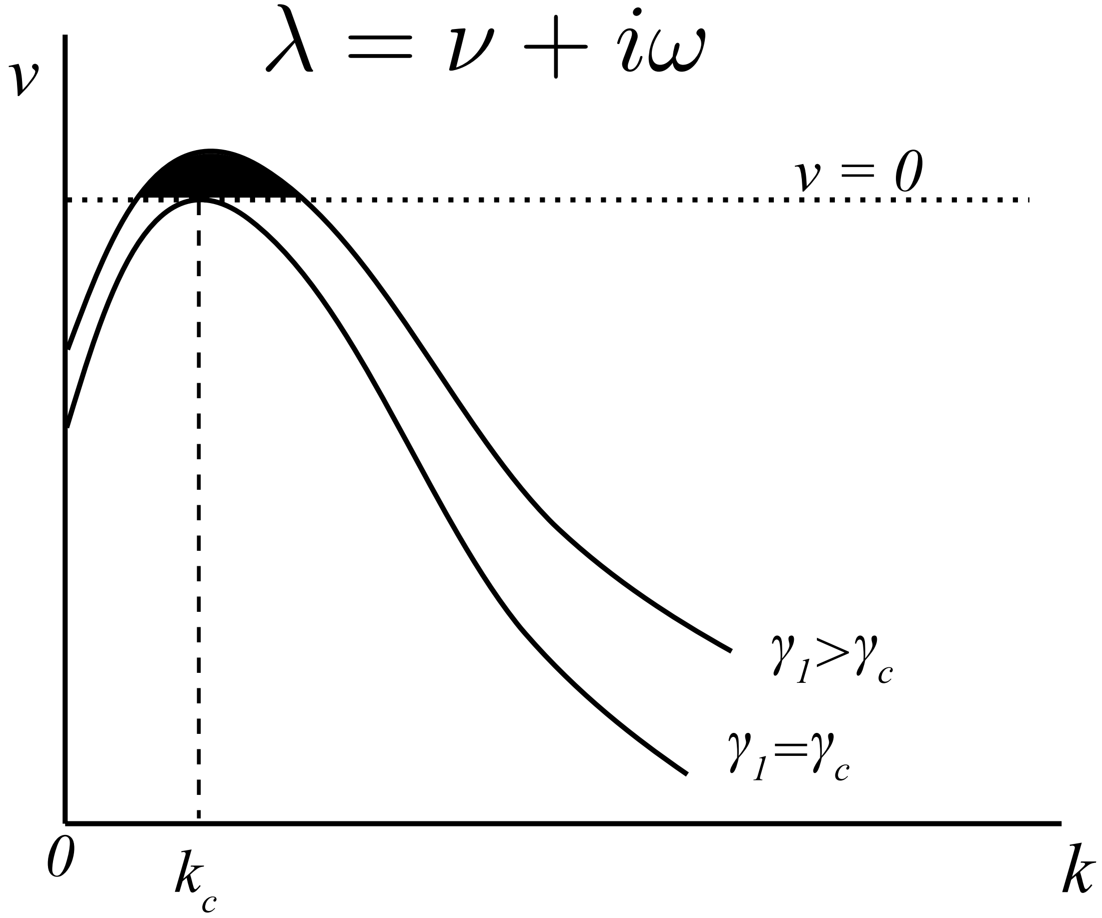
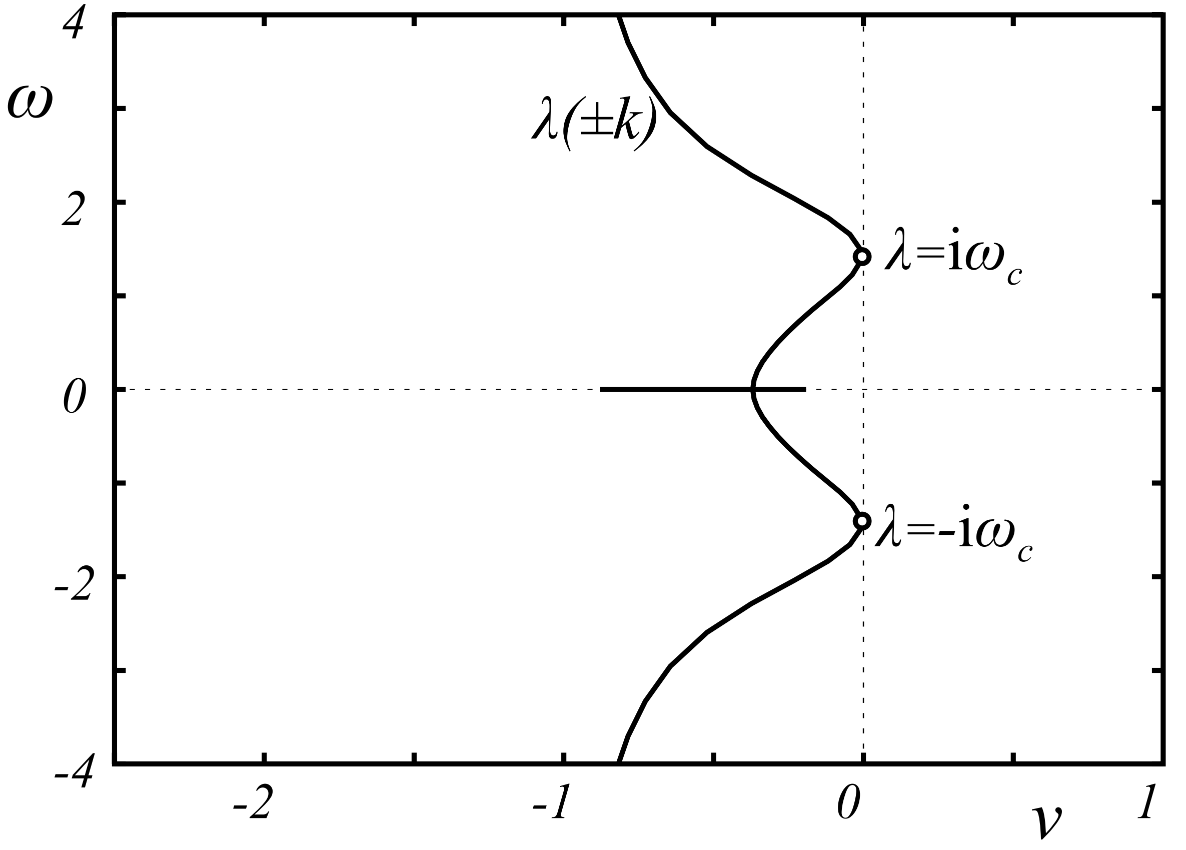
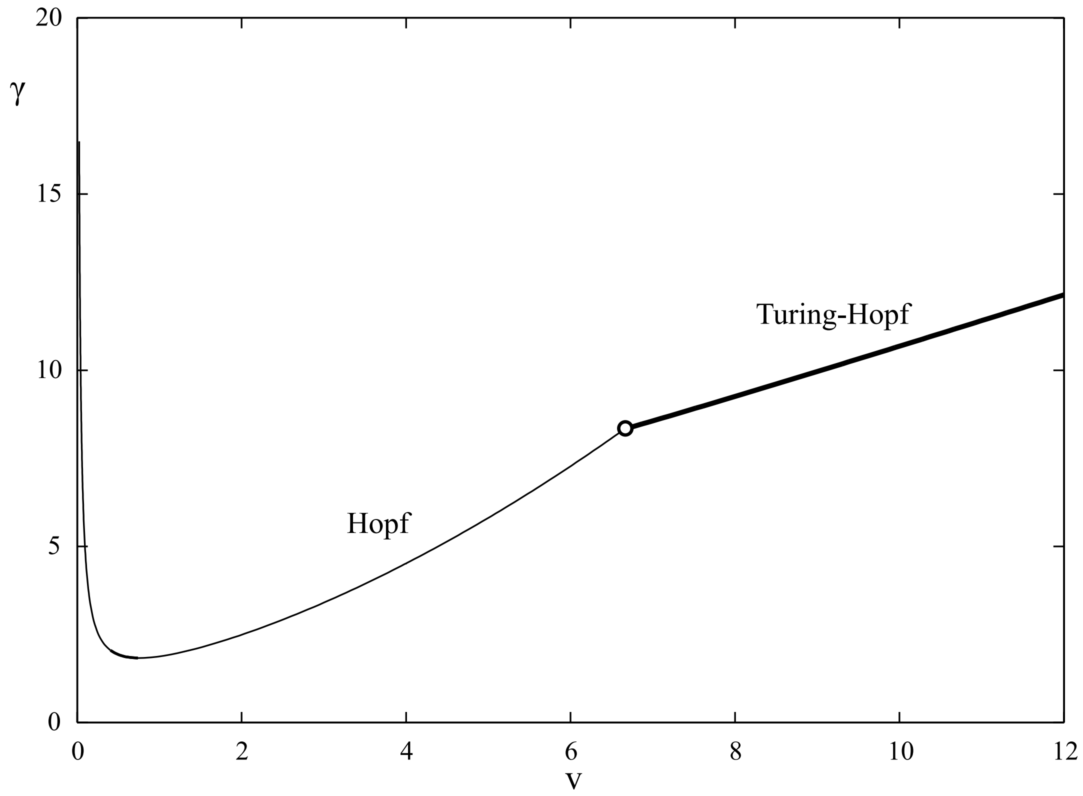
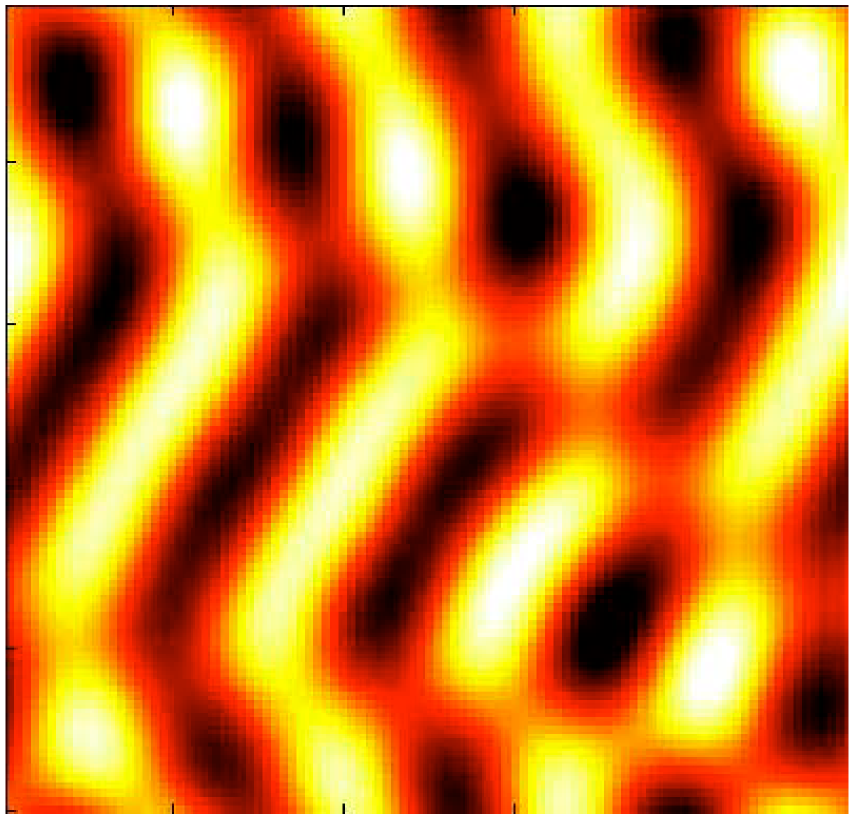


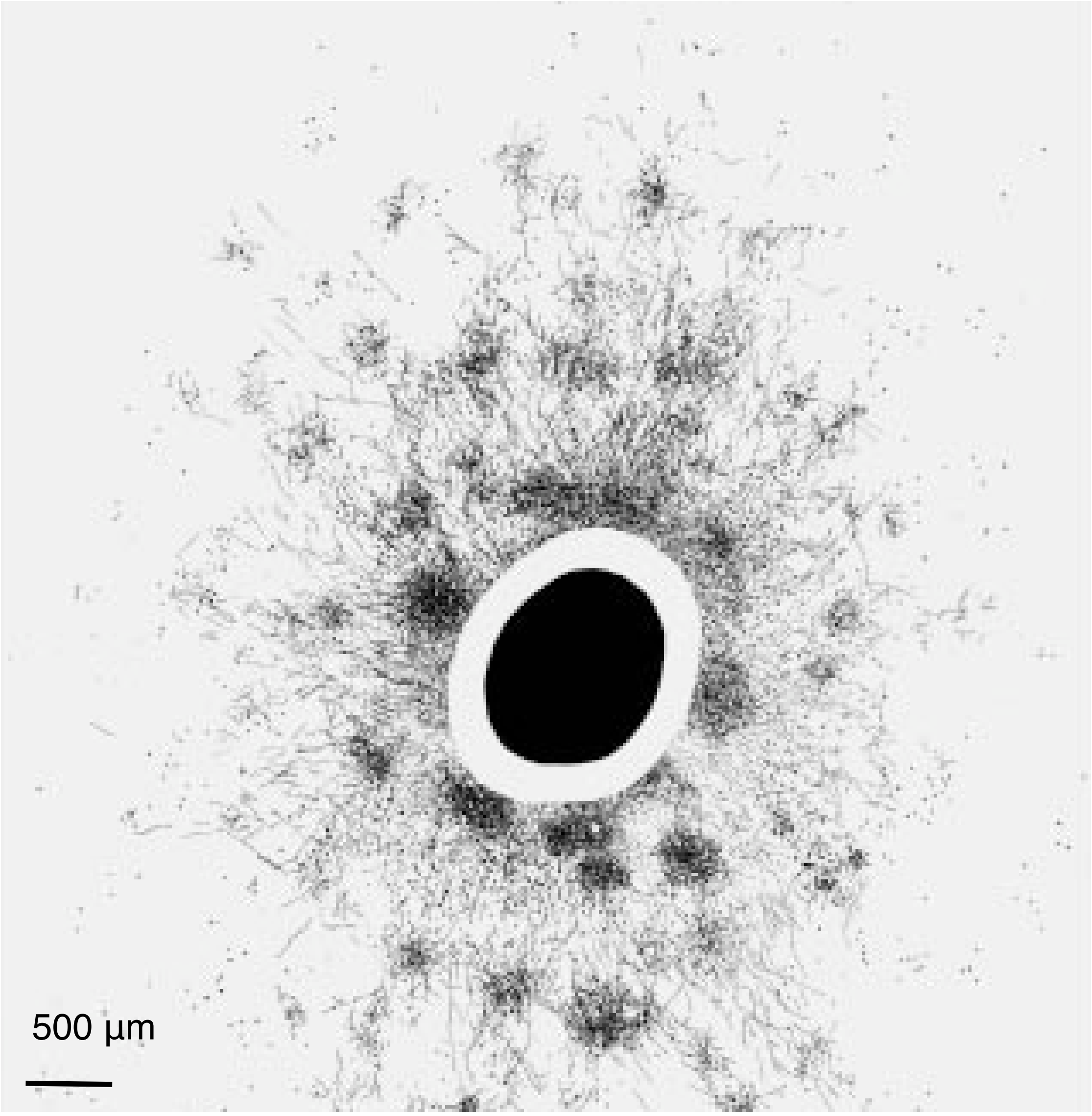






Spatial modulation

Neocortex has a crystalline micro-structure at the mm length scale, so that the assumption of isotropic connectivity has to be revised.



Anisotropy in a patchy connectional field: reconstruction of a tangential section through layers 2/3 of macaque area V1, showing a cholera toxin B tracer injection site and surrounding transported orthograde and retrograde label.

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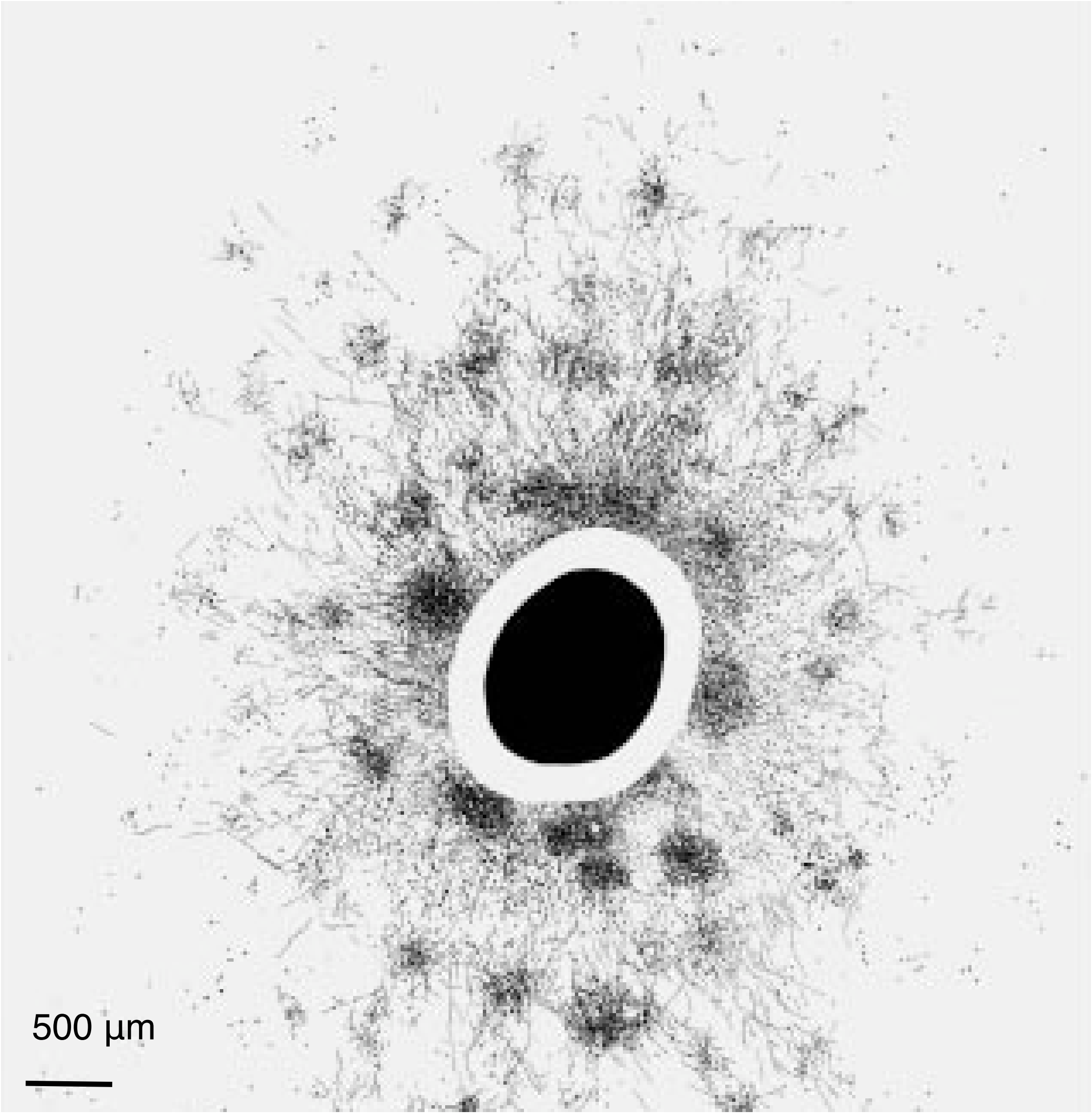

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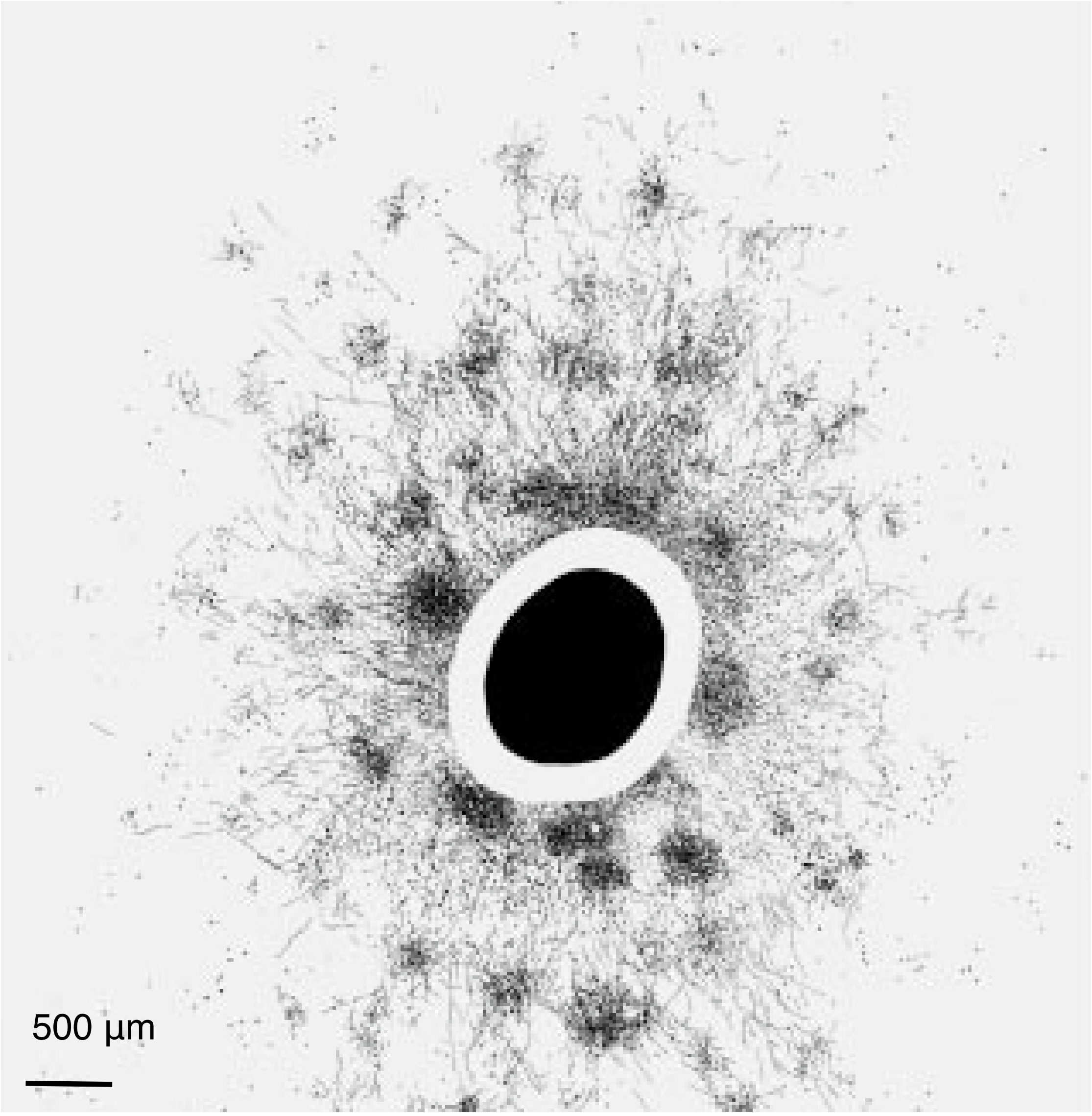

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Patchy propagators - P A Robinson, Physical Review E, 73:041904, 2006.

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same formal approach as before – λ now depends on the direction as well as the magnitude of $\mathbf{k}$

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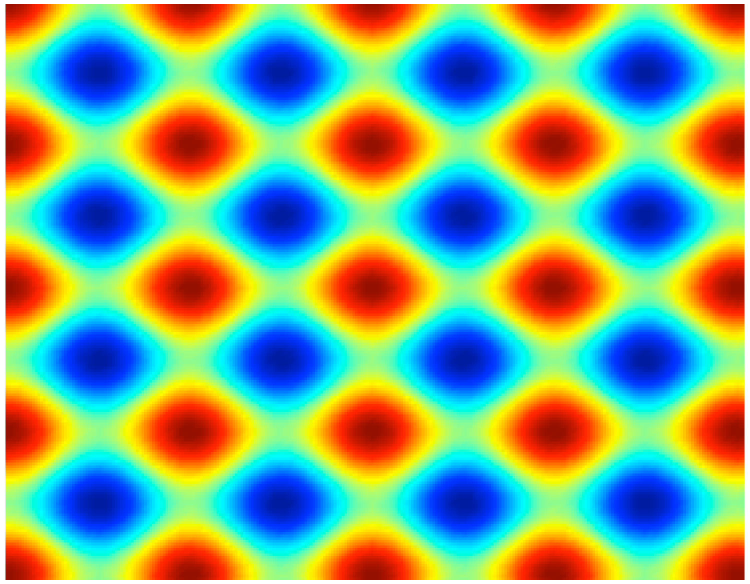

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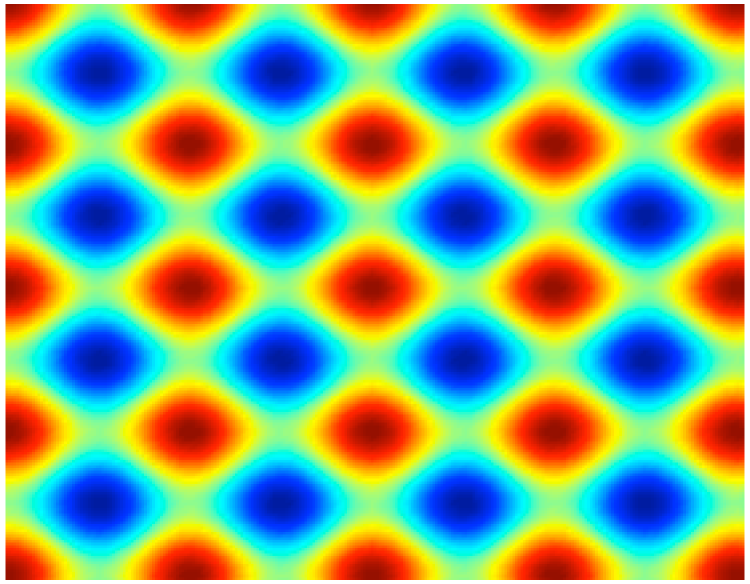

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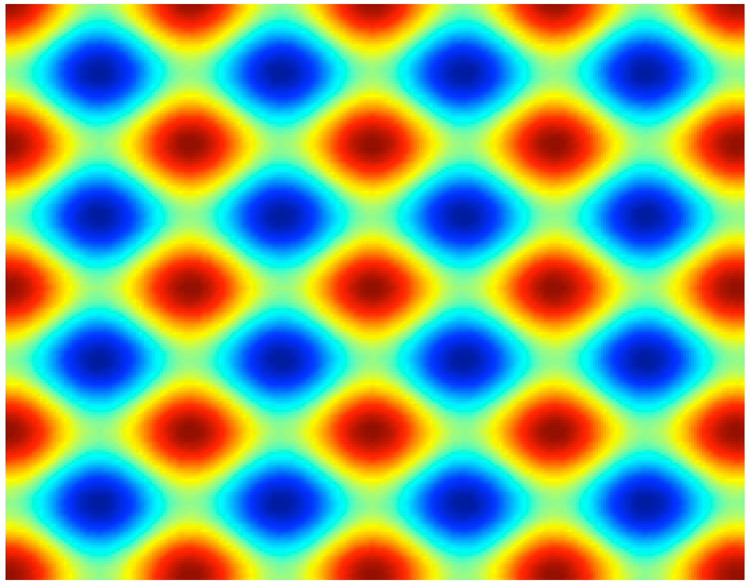

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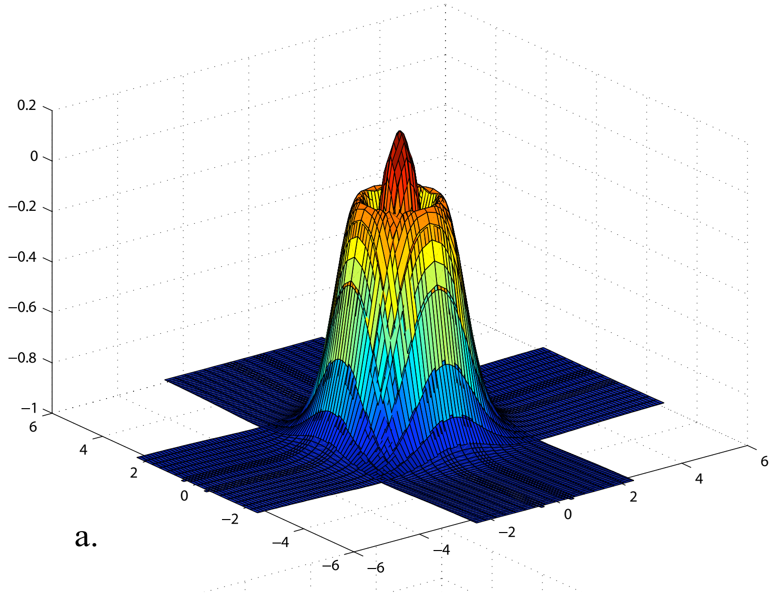

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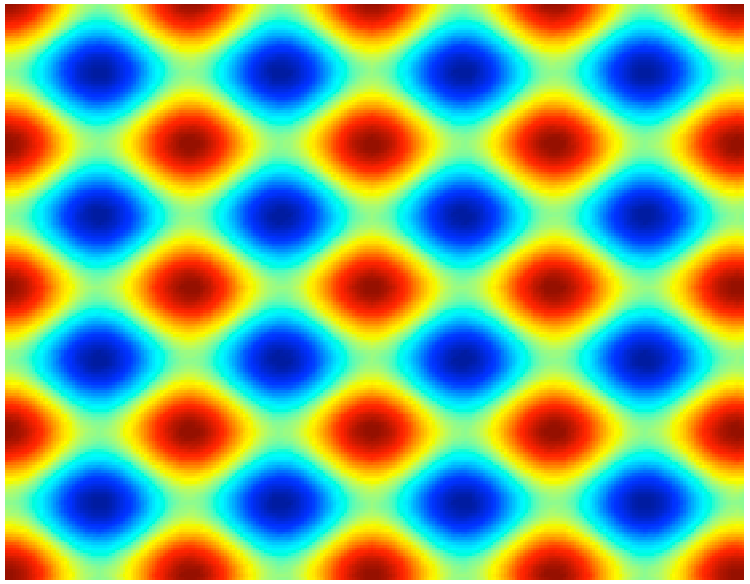

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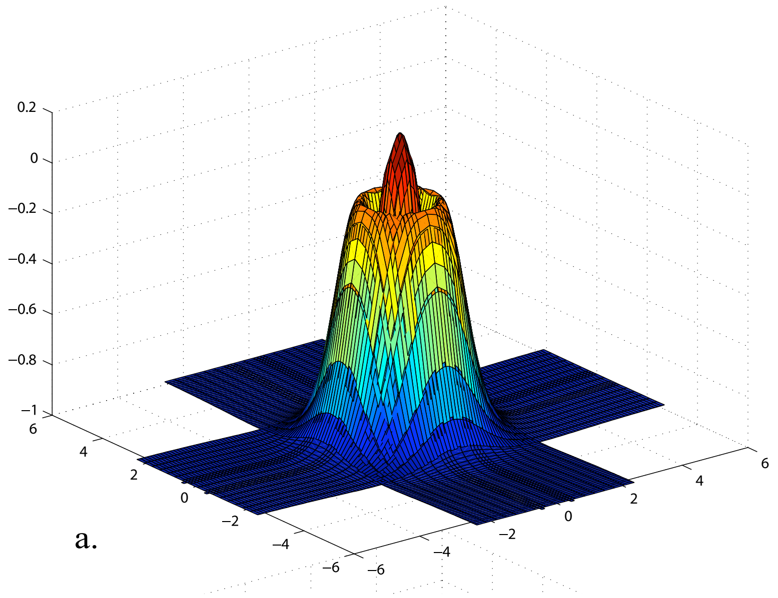
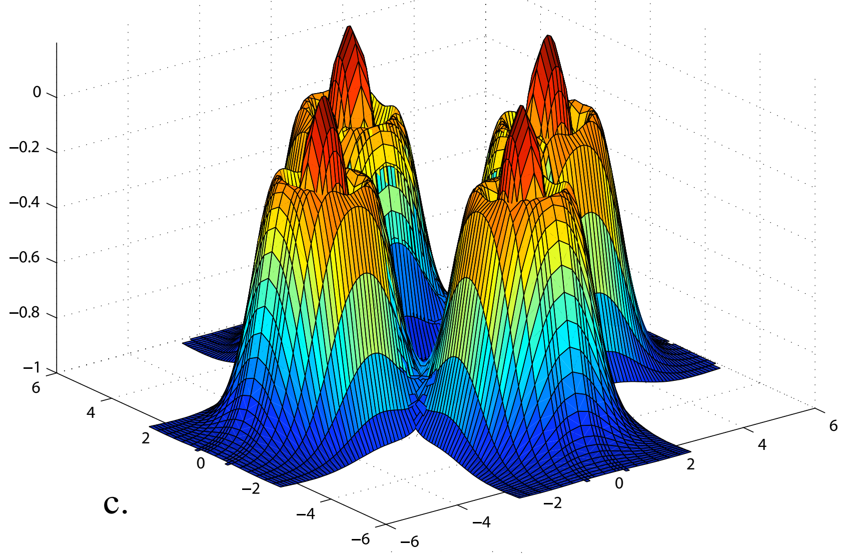


a.



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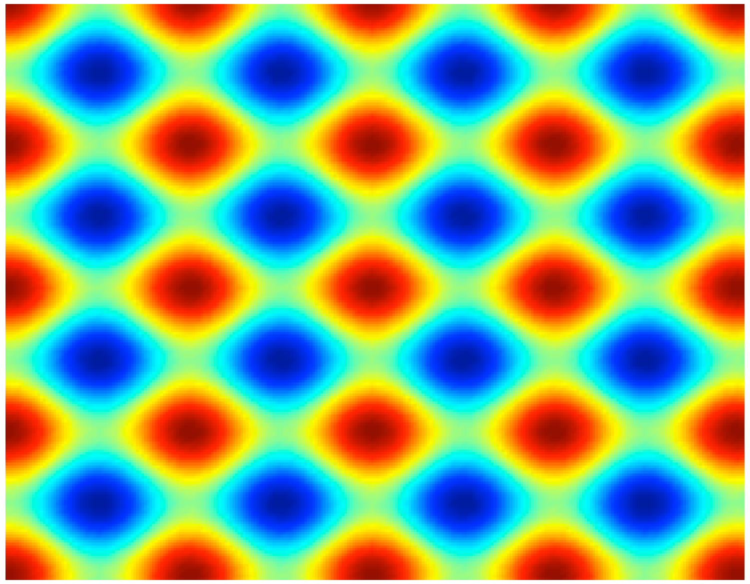

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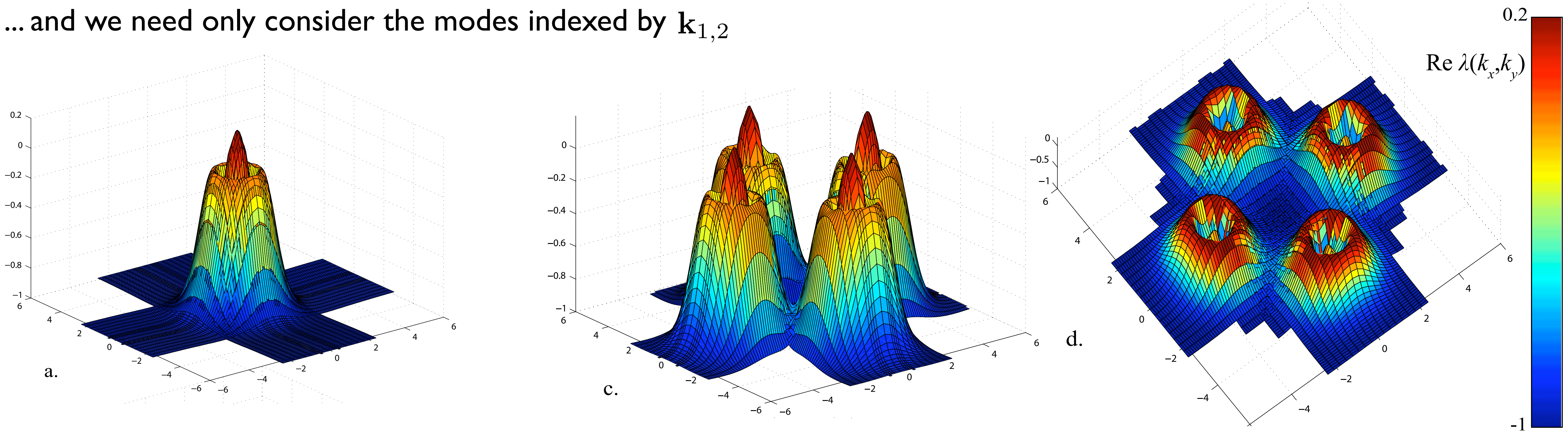
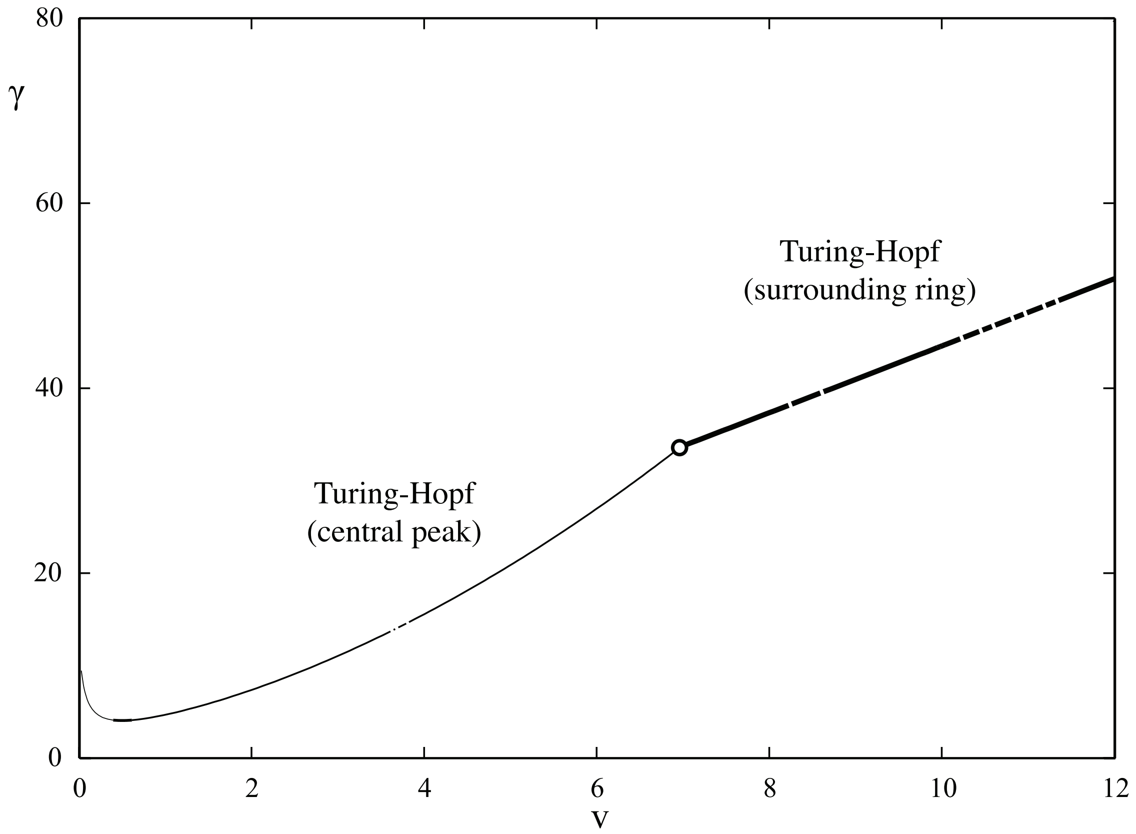
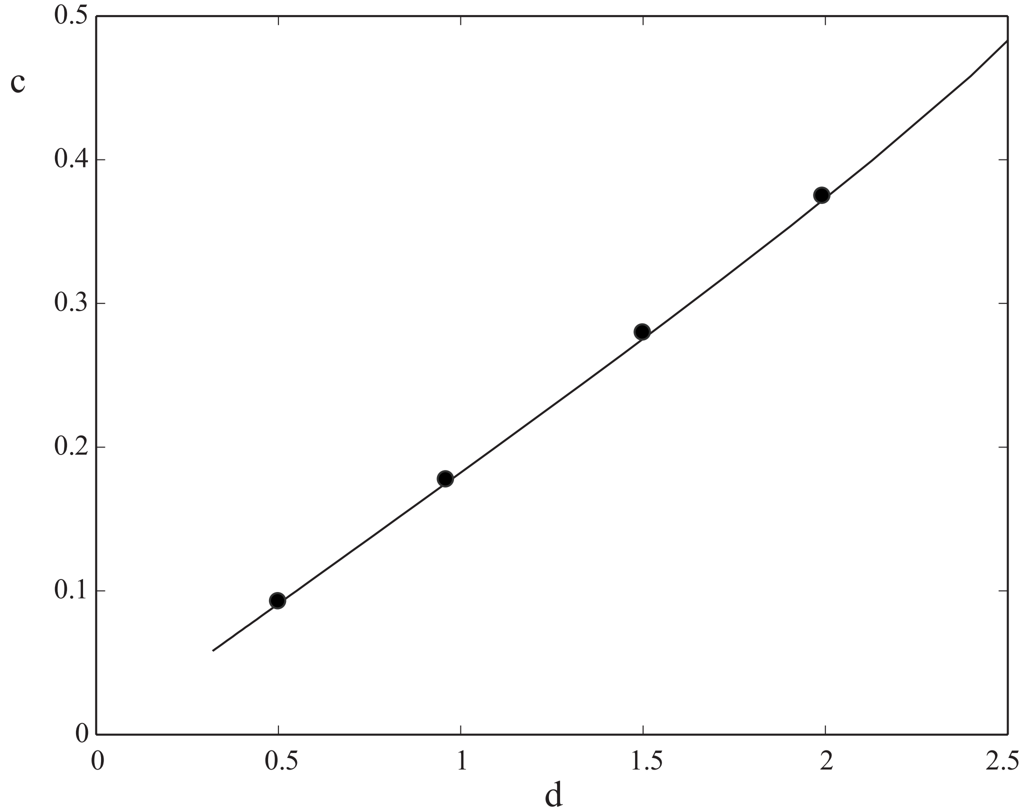
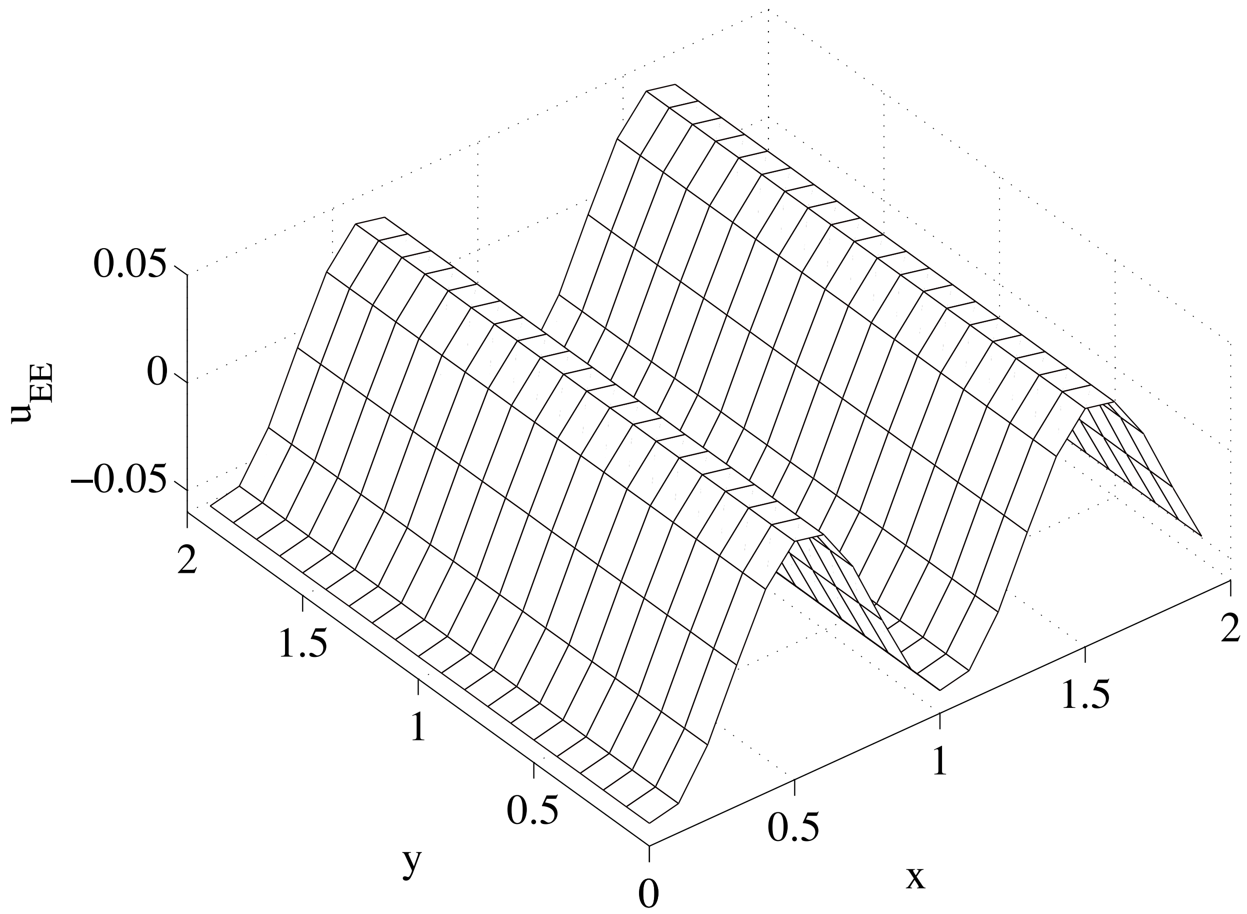
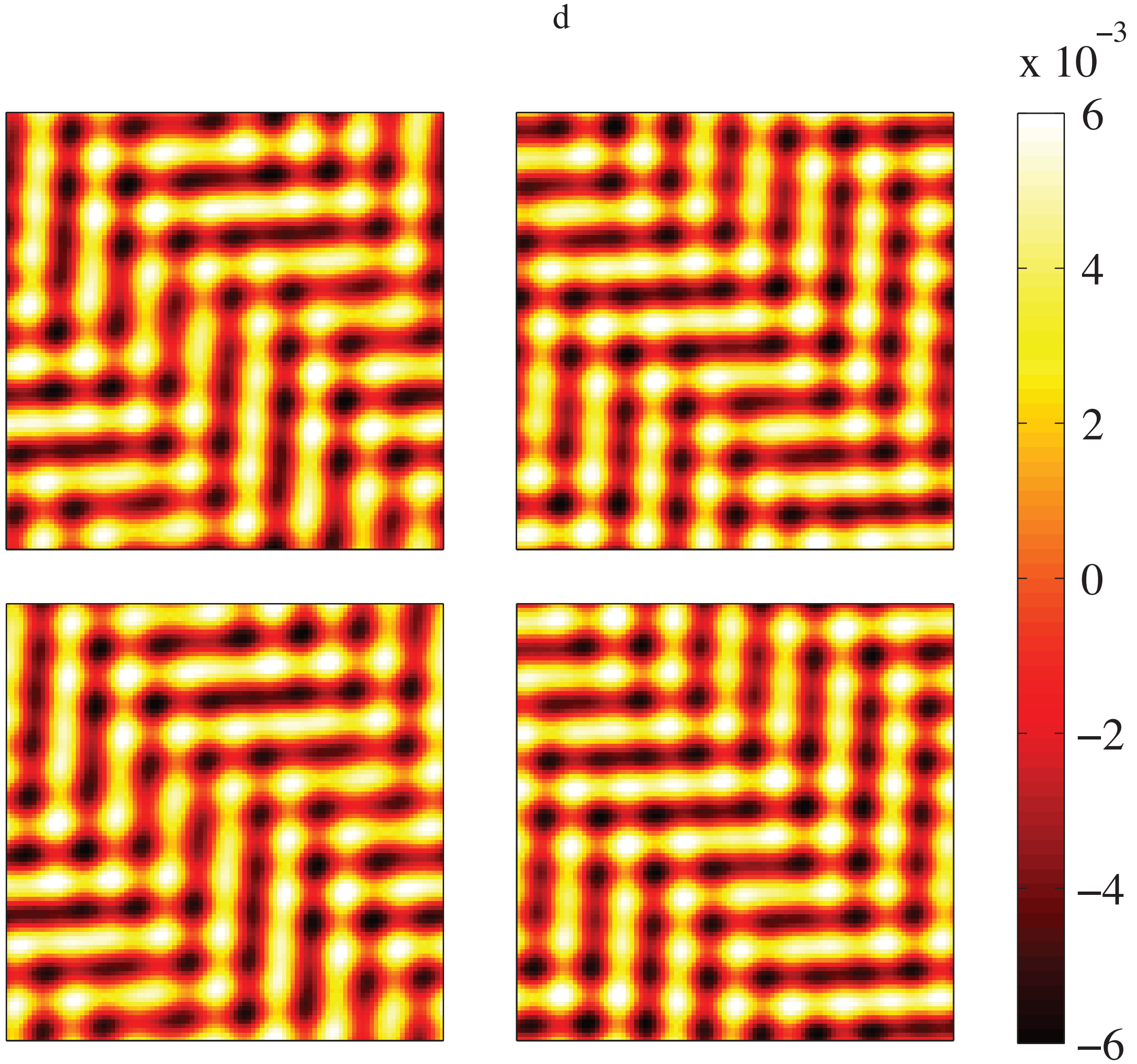
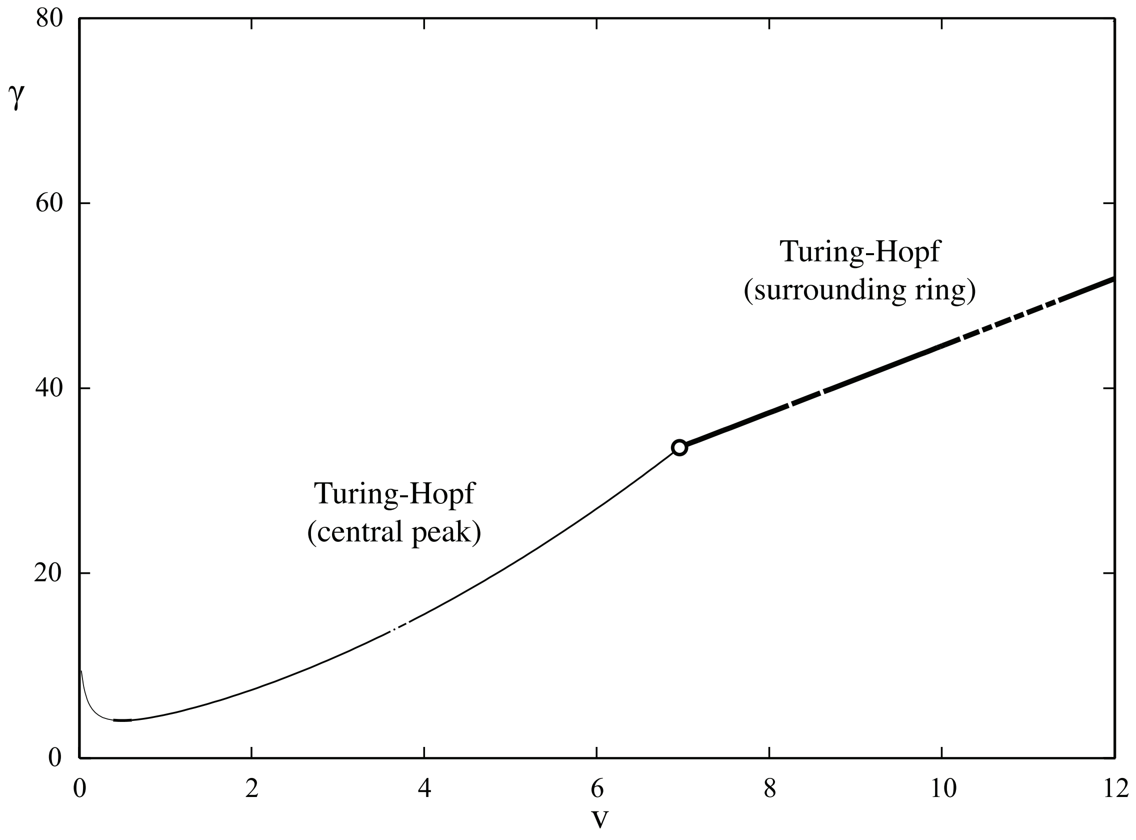



γ
80
60
40
20
0
Turing-Hopf
(surrounding ring)
Turing-Hopf
(central peak)
0
2
4
6
8
10
12
v


c
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1
1.5
2
2.5
d


0.05
0
−0.05
u_{EE}
2
1.5
1
0.5
y
0
0.5
1
1.5
2
x


x 10^{-3}
6
4
2
0
−2
−4
−6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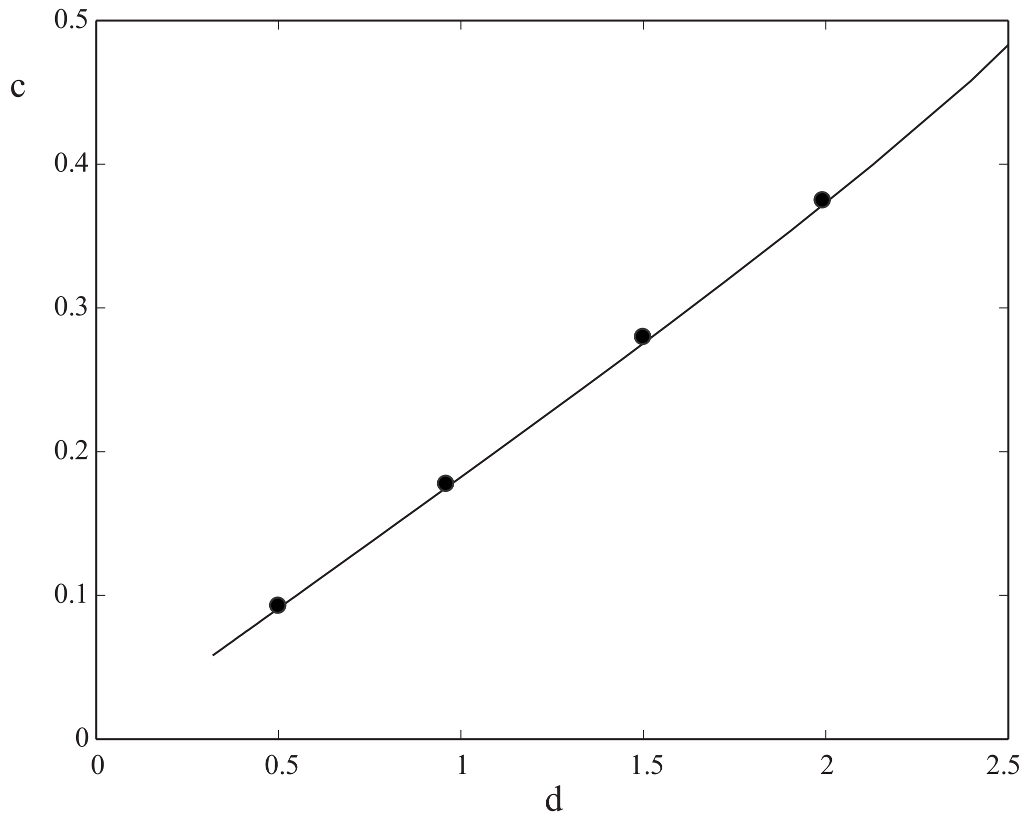
c
0.5
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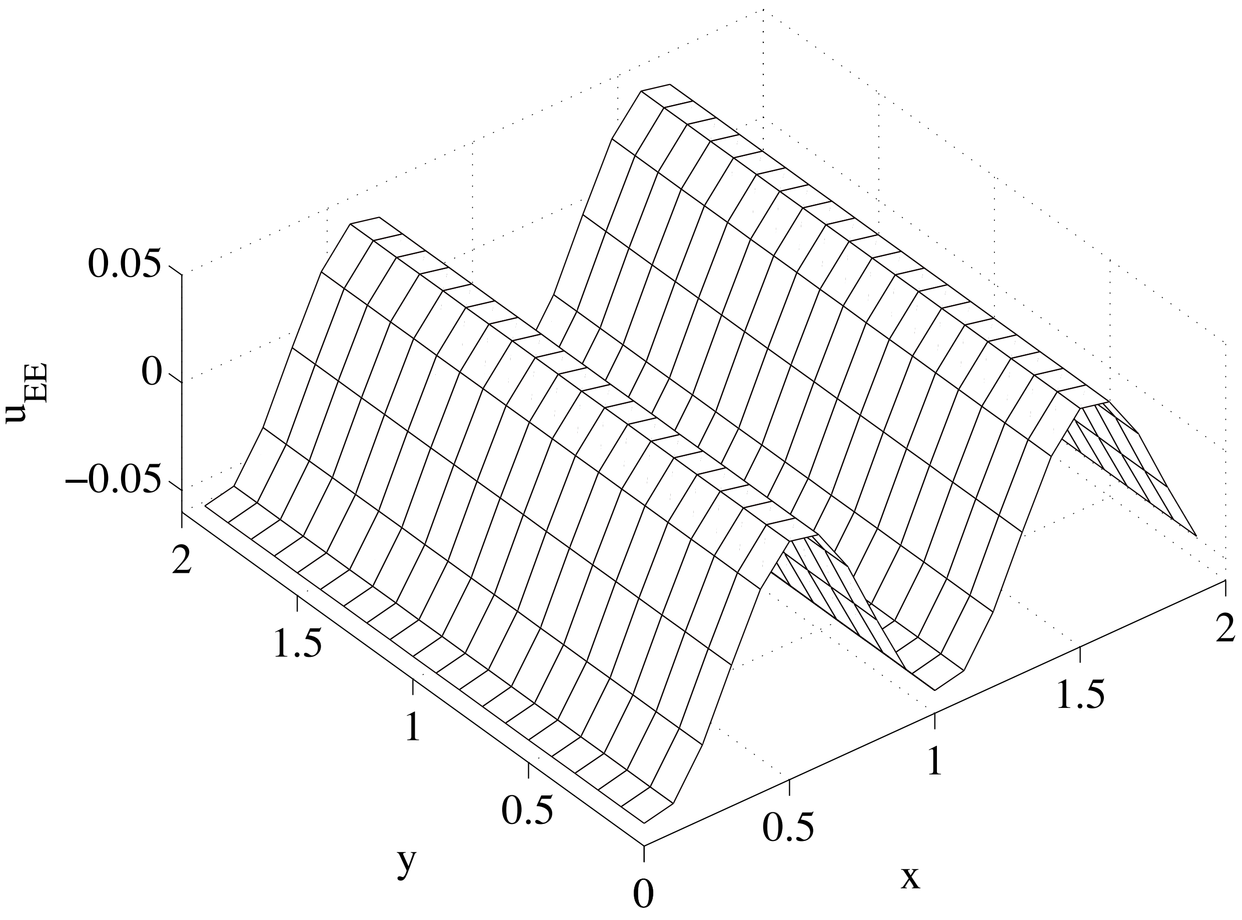
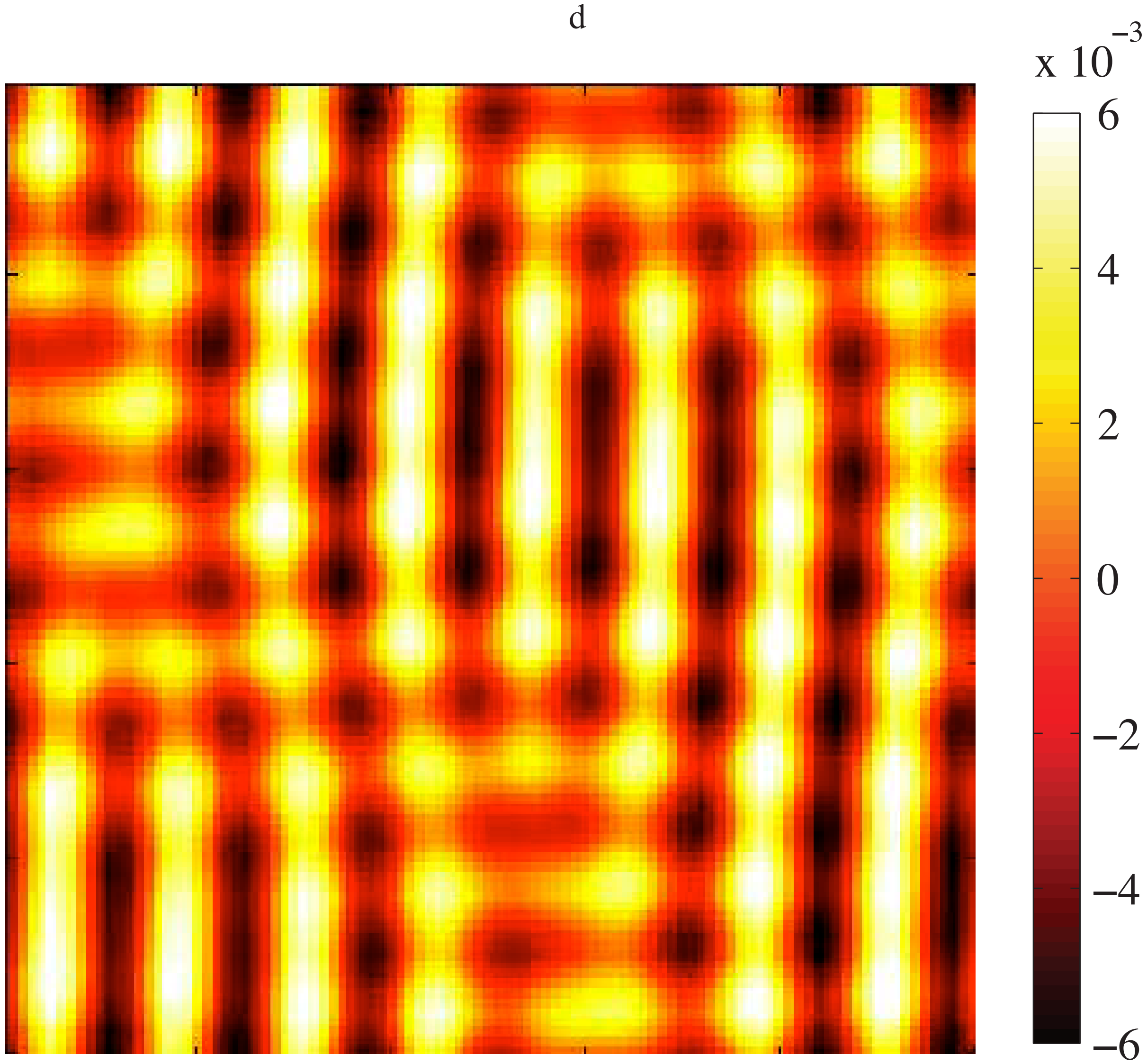

u_{EE}
0.05
0
−0.05
2
1.5
1
0.5
y
0
0.5
1
1.5
2
x


x 10^{-3}
6
4
2
0
−2
−4
−6

Threshold Accommodation

$$u(x,t) = \int_{-\infty}^{\infty} w(x-y) \int_{-\infty}^{t} \eta(t-s) f(u(y,s)-h) \mathrm{d}s\mathrm{d}y$$

Hill (1936), “... the threshold rises when the *local potential* is maintained ... and reverts gradually to its original value when the nerve is allowed to rest.”

$$h_t = -(h - h_0) + \kappa H(u - \theta)$$

Time-independent solutions $(u, h) = (q(x), p(x))$

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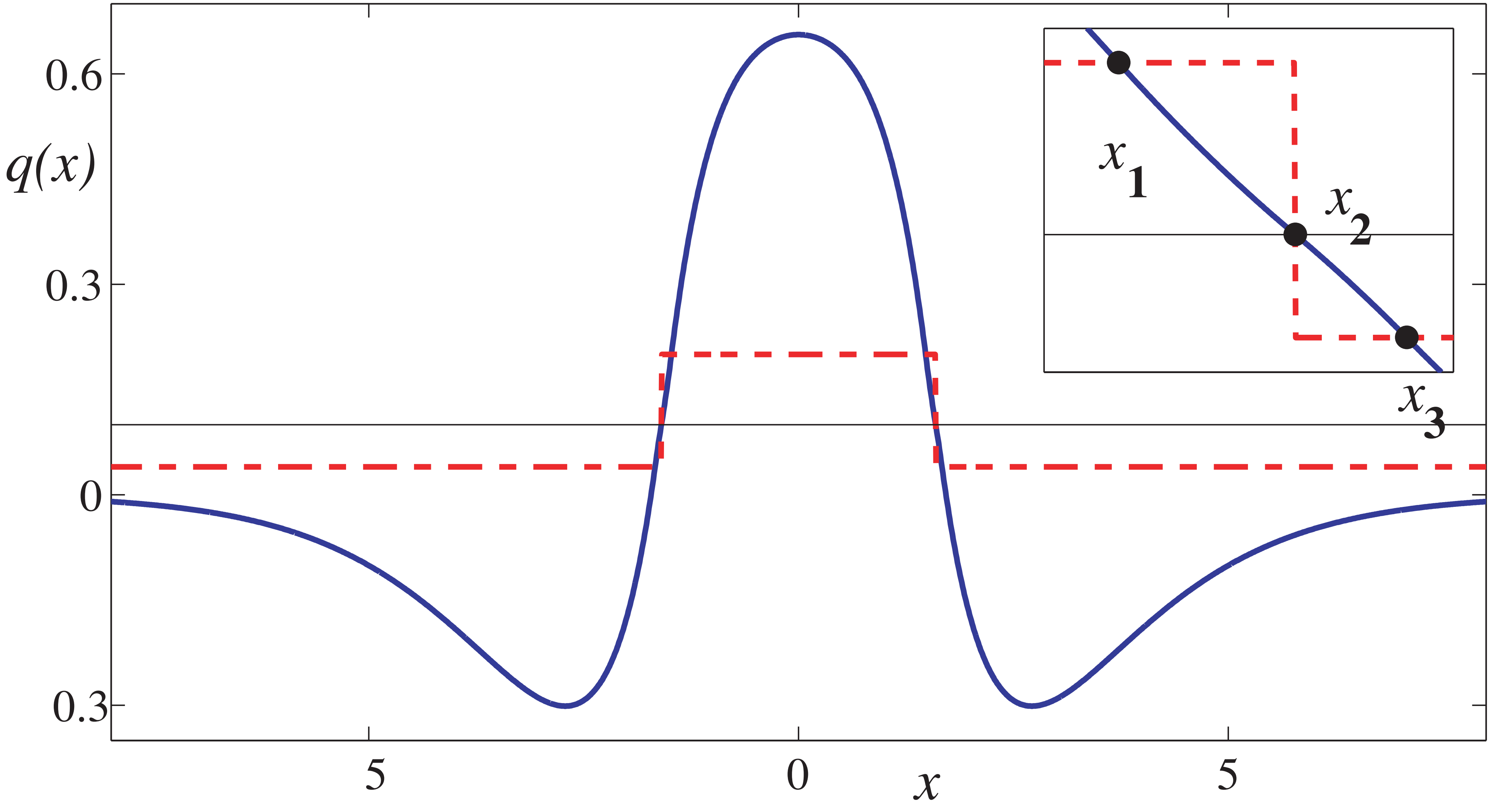
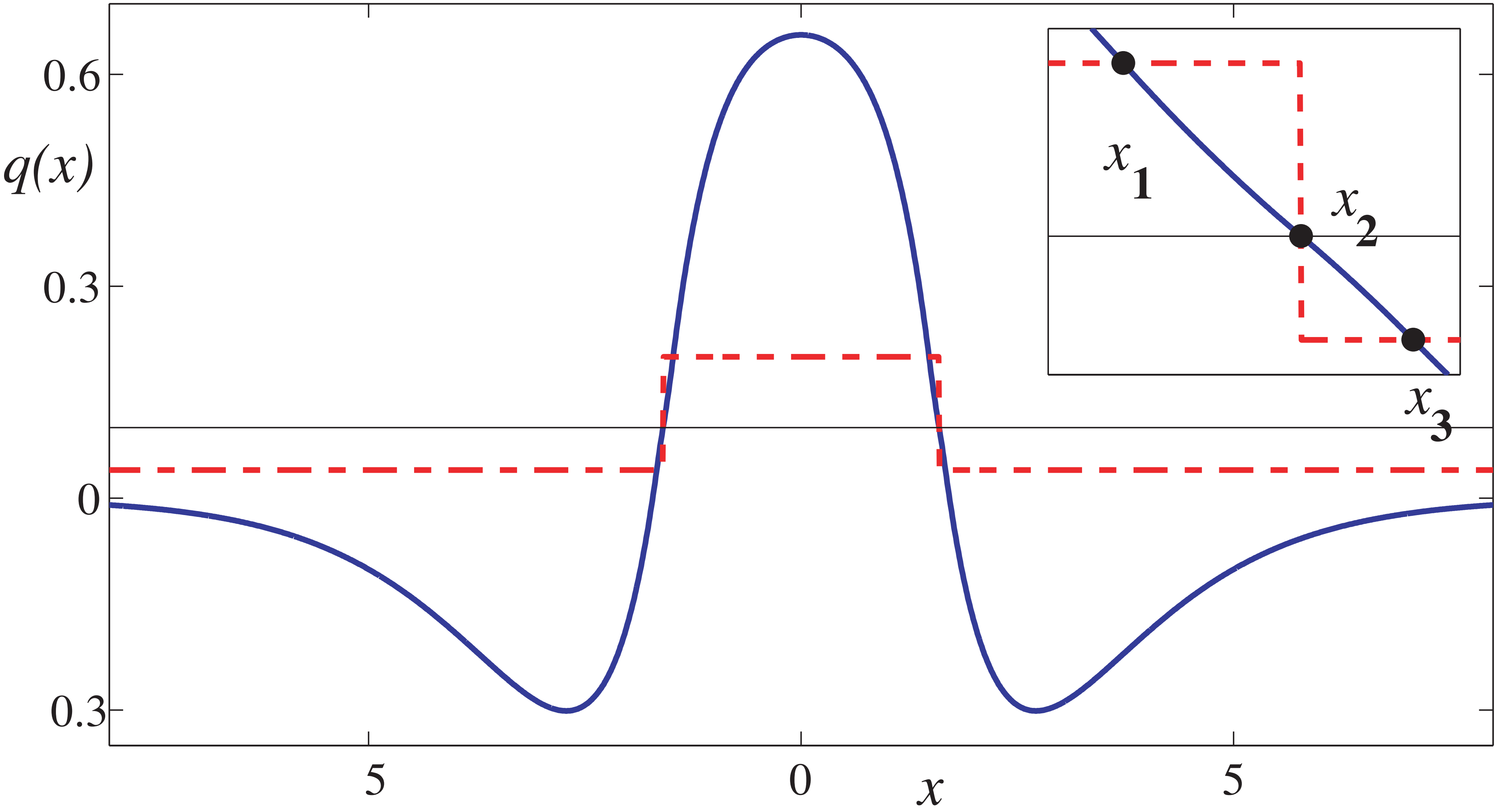


Time-independent solutions $(u,h)=(q(x),p(x))$

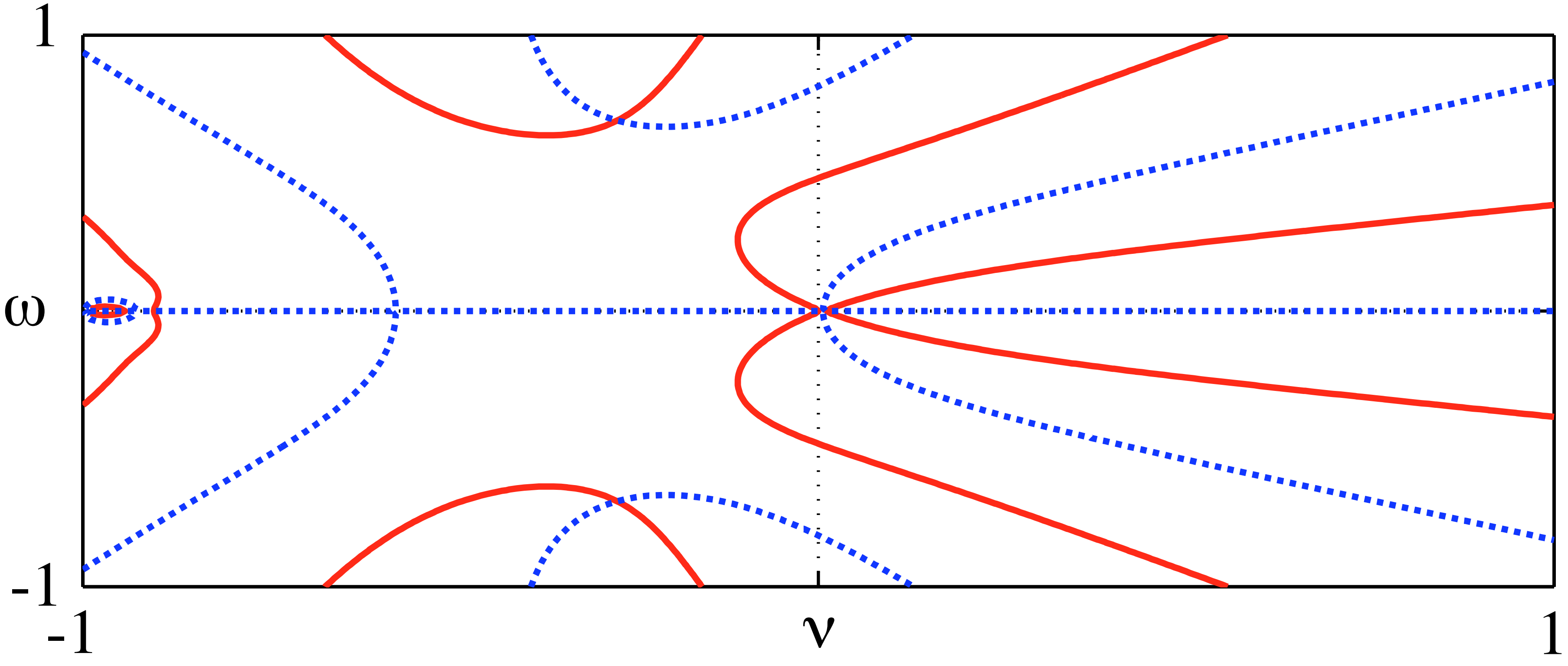
$$(w \otimes f)(x,t) = \int_{-\infty}^{\infty} w(y) f(x-y,t)\,\mathrm{d}y$$

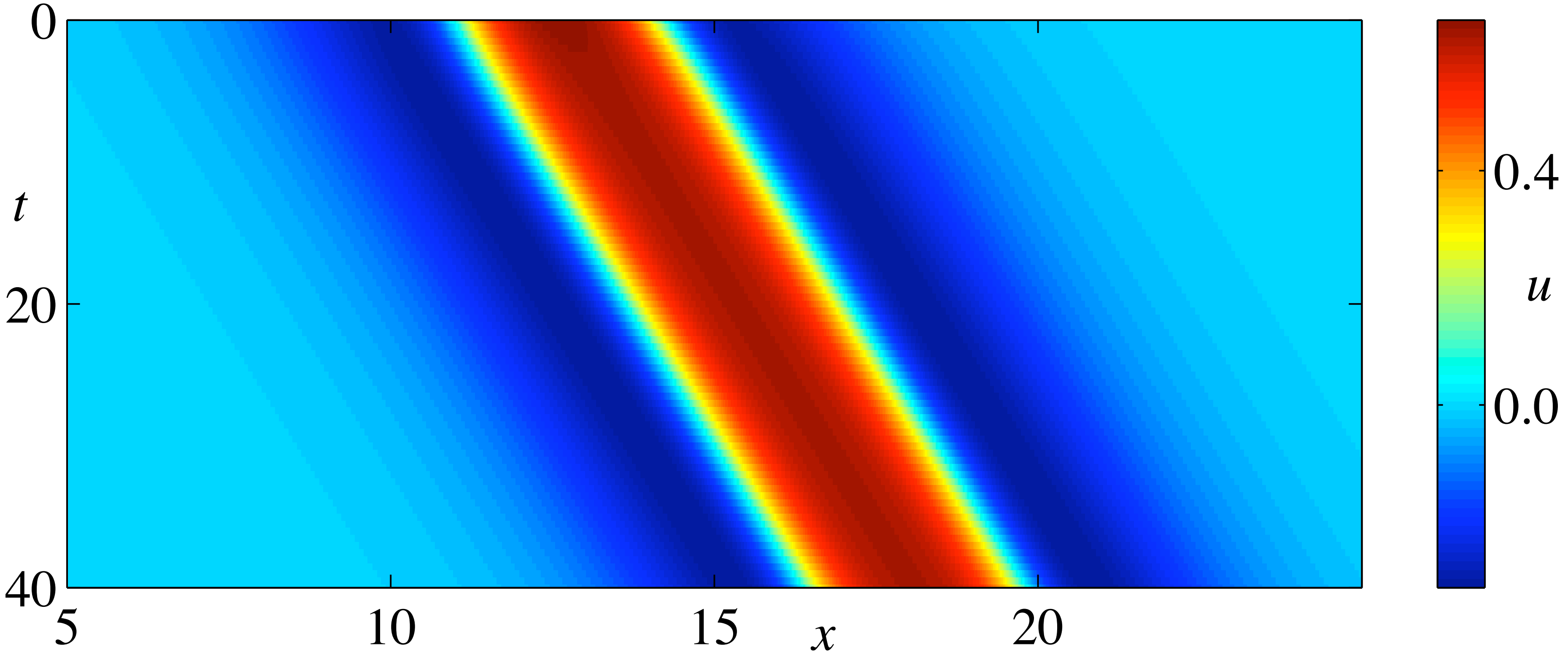
$$q = w \otimes H(q-p), \qquad p = \begin{cases} h_0 & q < \theta \\ h_0 + \kappa & q \geq \theta \end{cases}$$



Bump Stability III: $\eta(t) = \alpha \mathrm{e}^{-\alpha t}$

Low κ instability on Re axis (increasing α)





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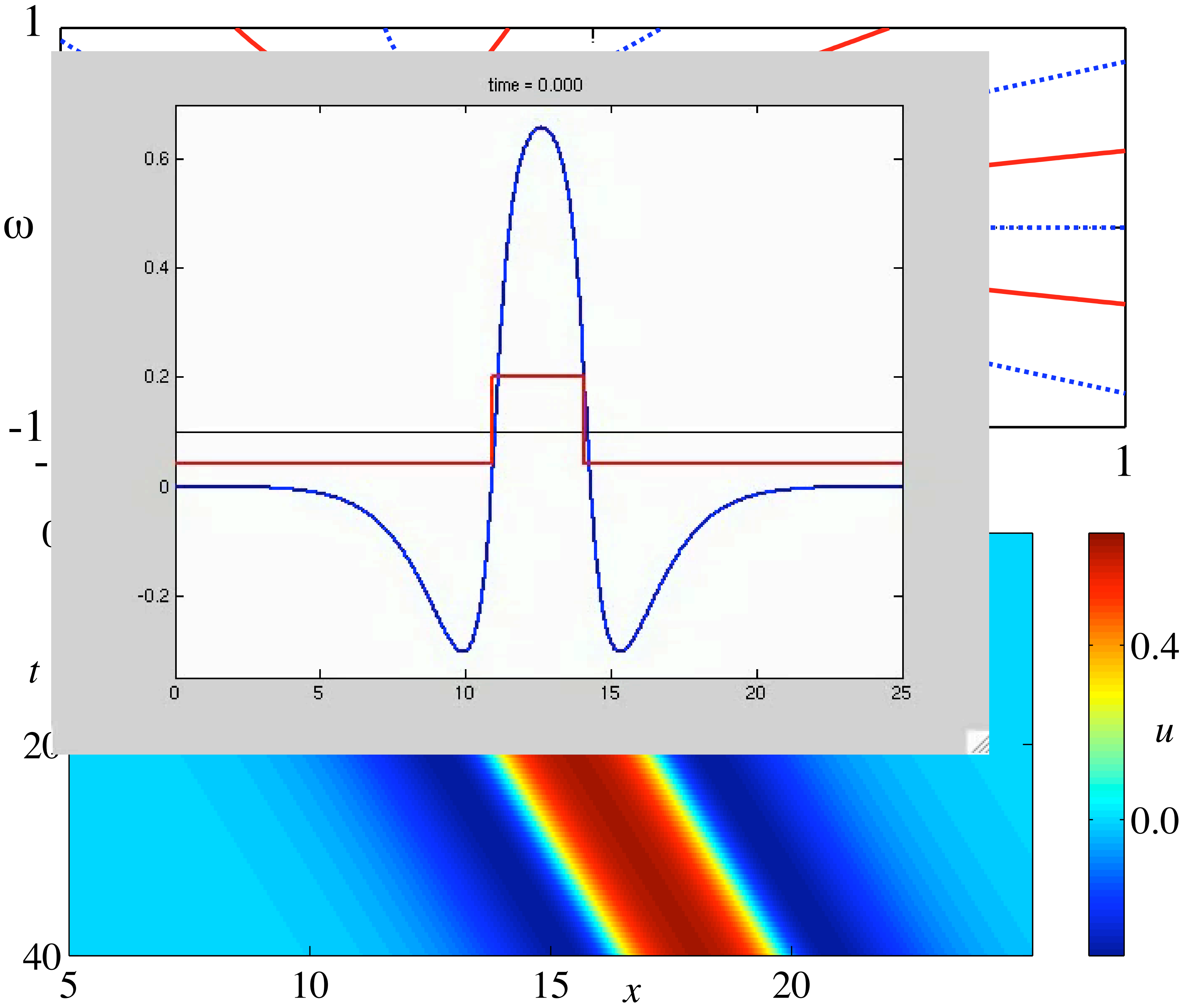
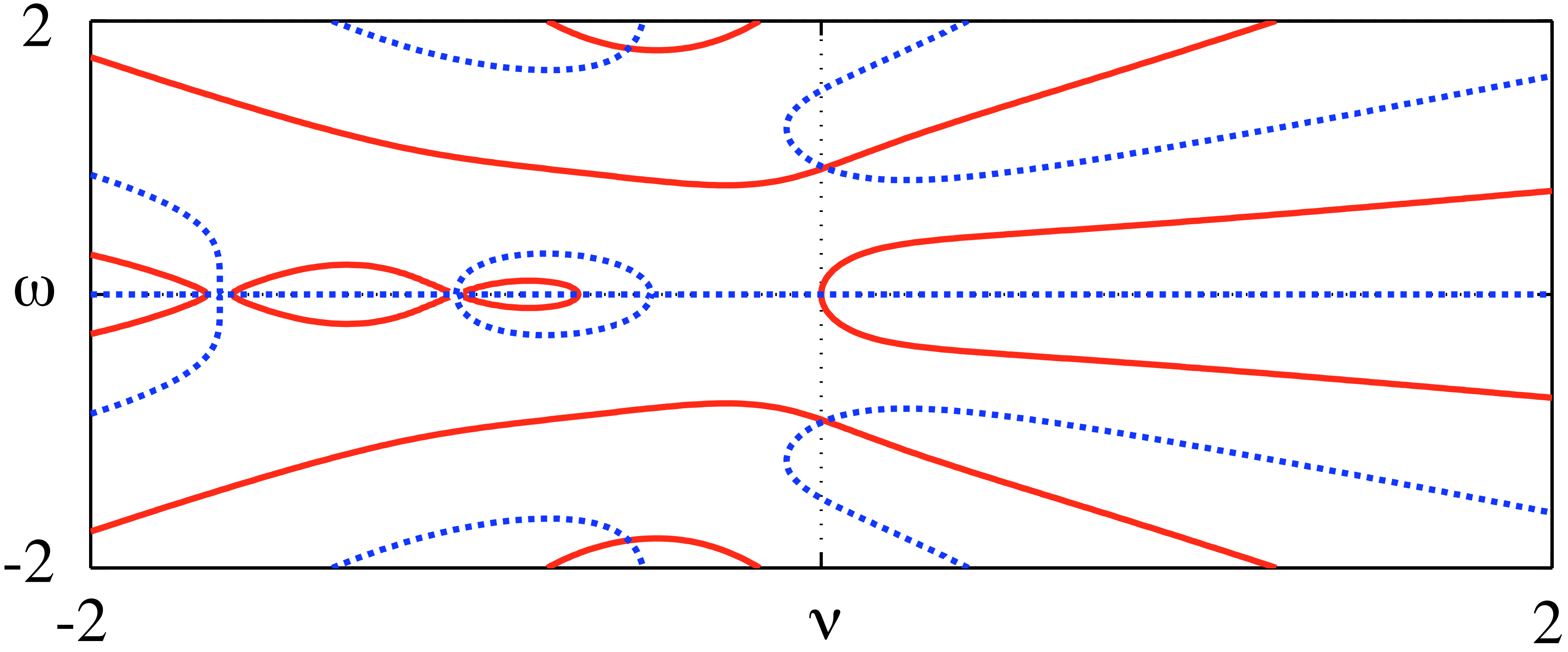
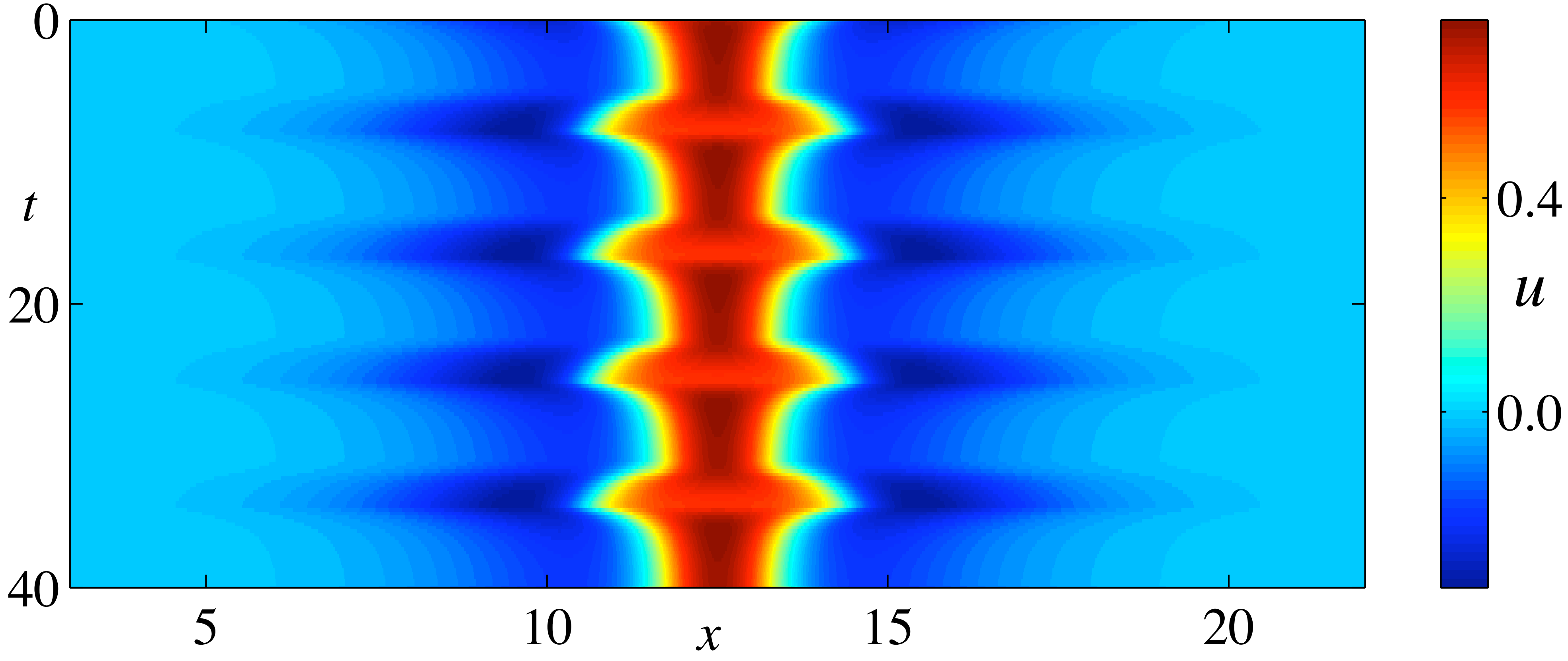


Bump Stability IV

High κ instability on Im axis (increasing α) gives a breather





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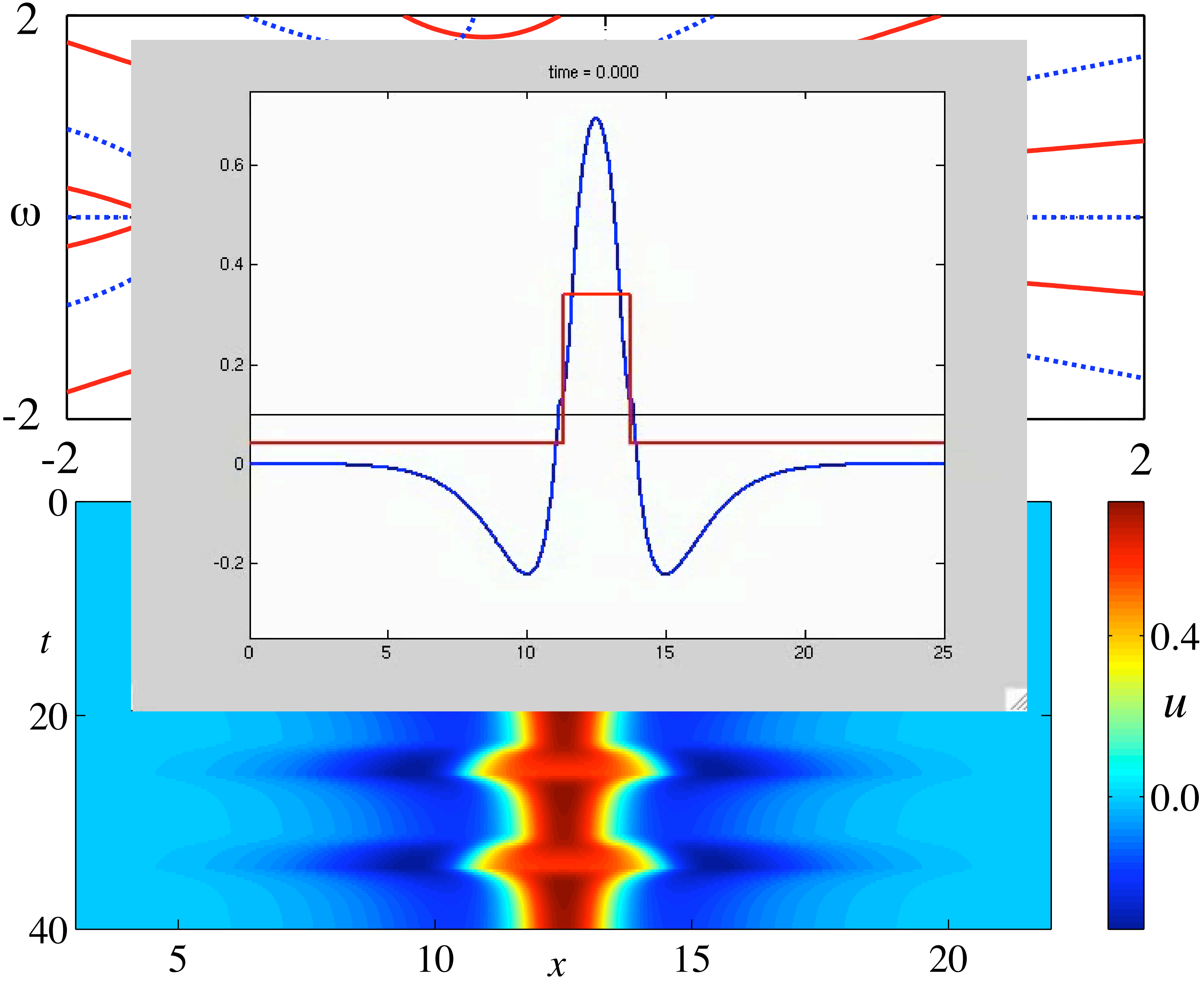
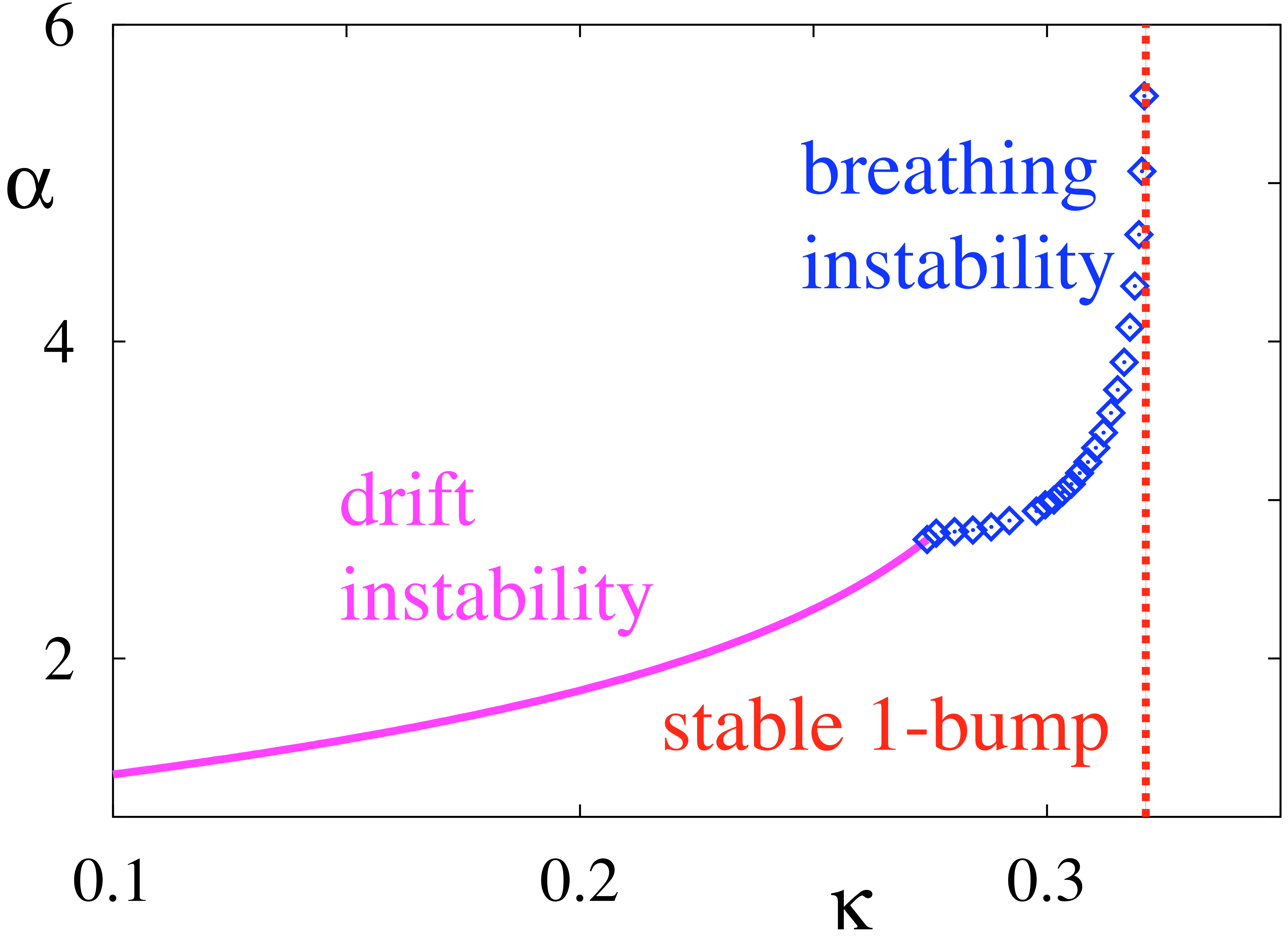
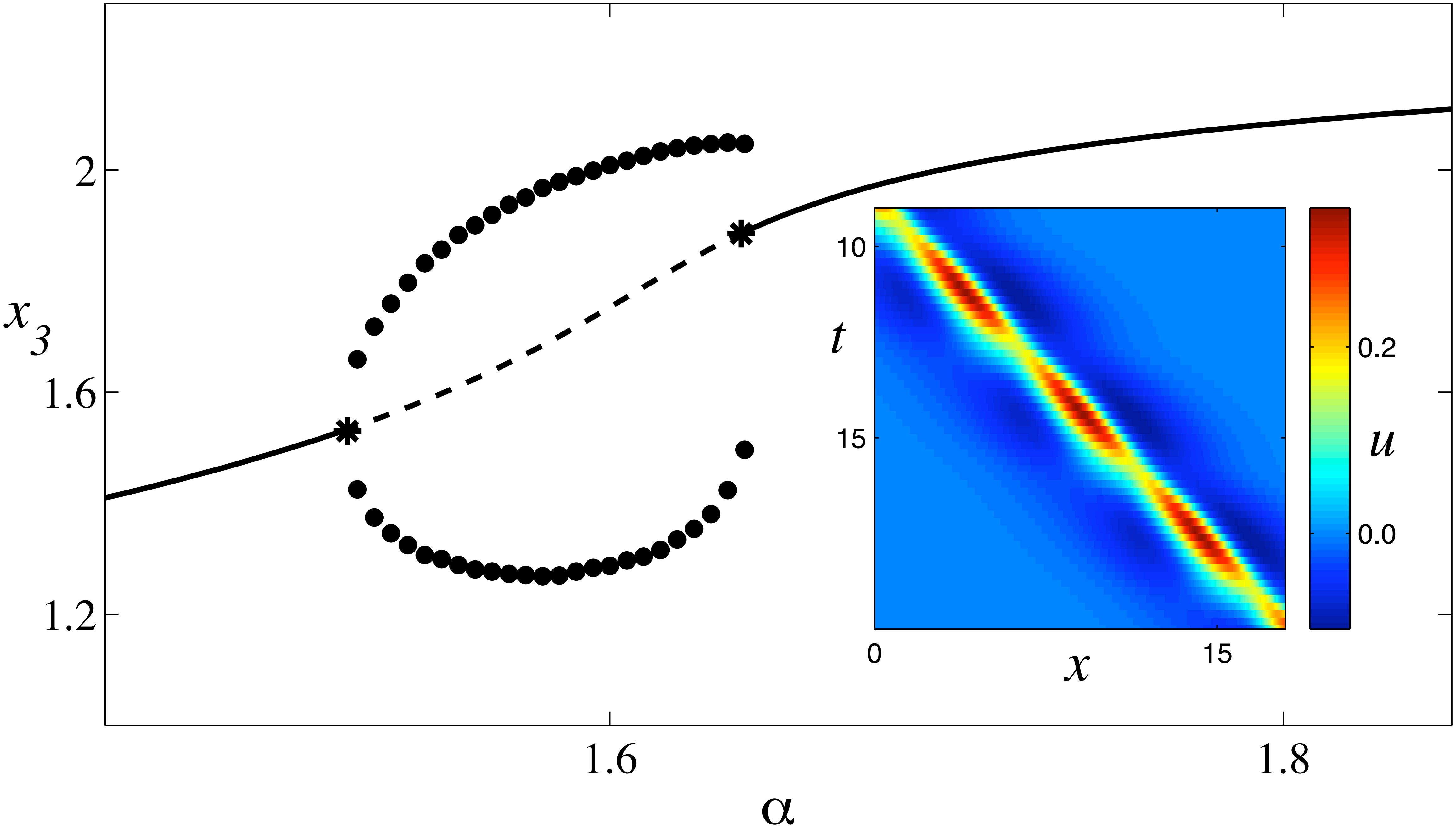


Summary of Bump instabilities



Dynamic instability of pulses

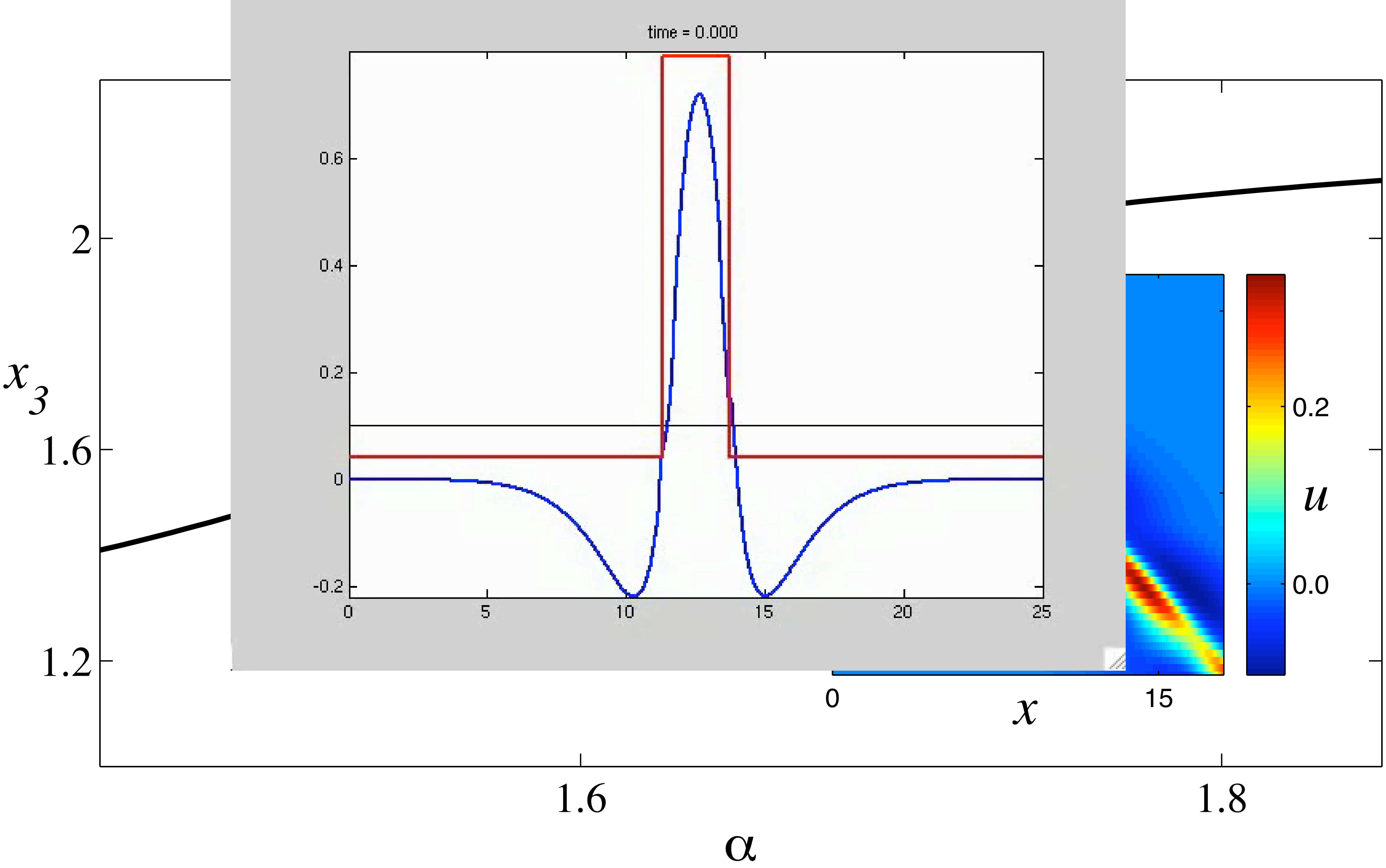
A pair of complex conjugate eigenvalues crosses the Im axis at $\alpha \approx 1.52, \alpha \approx 1.64$



Exact solution (curve); Hopf bifurcations (asterisks); Numerics (points).

Dynamic instability of pulses

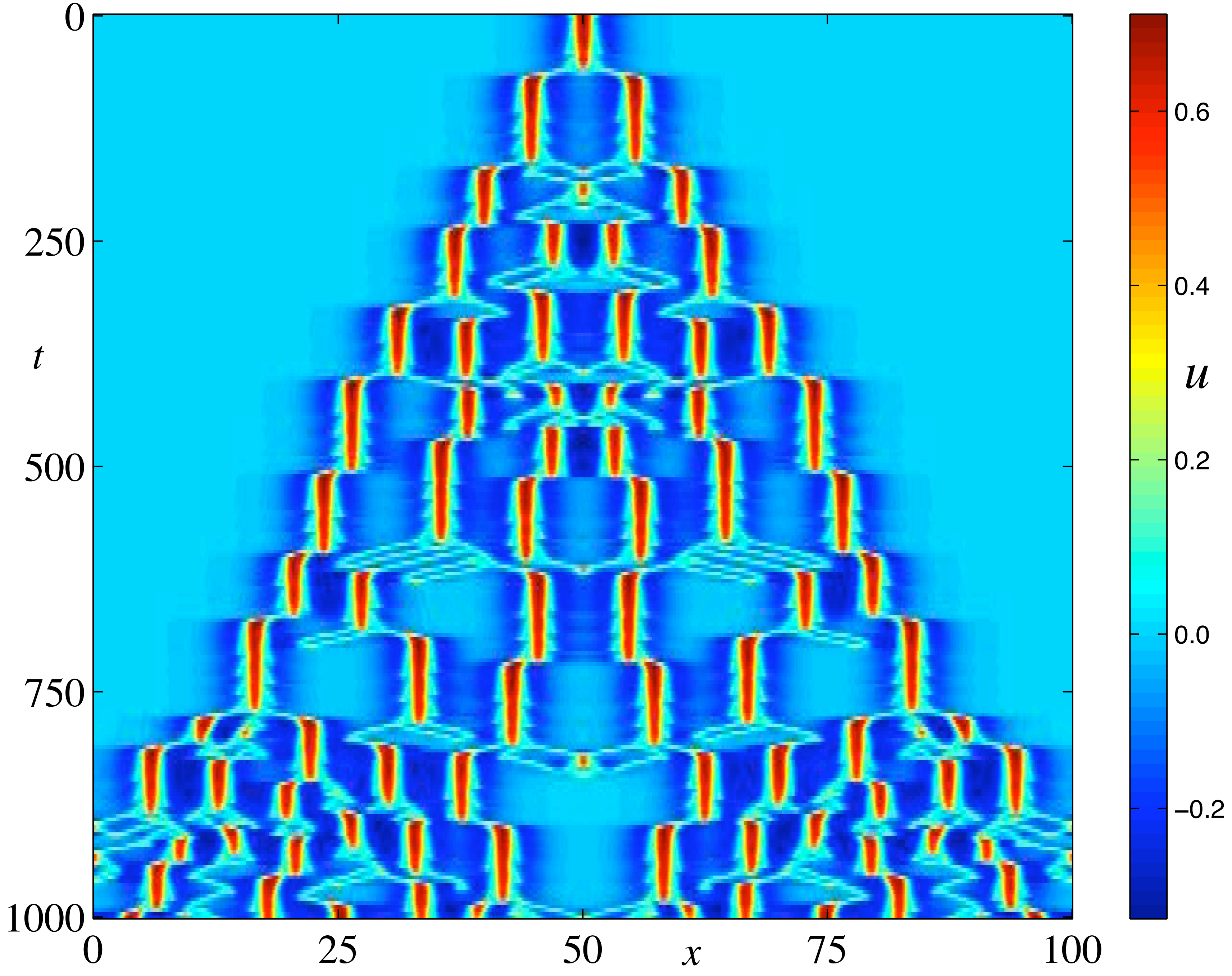
A pair of complex conjugate eigenvalues crosses the Im axis at $\alpha \approx 1.52, \alpha \approx 1.64$



Exact solution (curve); Hopf bifurcations (asterisks); Numerics (points).

Exotic Dynamics

... including asymmetric breathers, multiple bumps, multiple pulses, periodic traveling waves, and bump-splitting instabilities that appear to lead to spatio-temporal chaos.



S Coombes and M R Owen: Bumps, breathers and waves in a neural network with spike frequency adaptation. PRL, 94, 148102, (2005).

Exotic Dynamics

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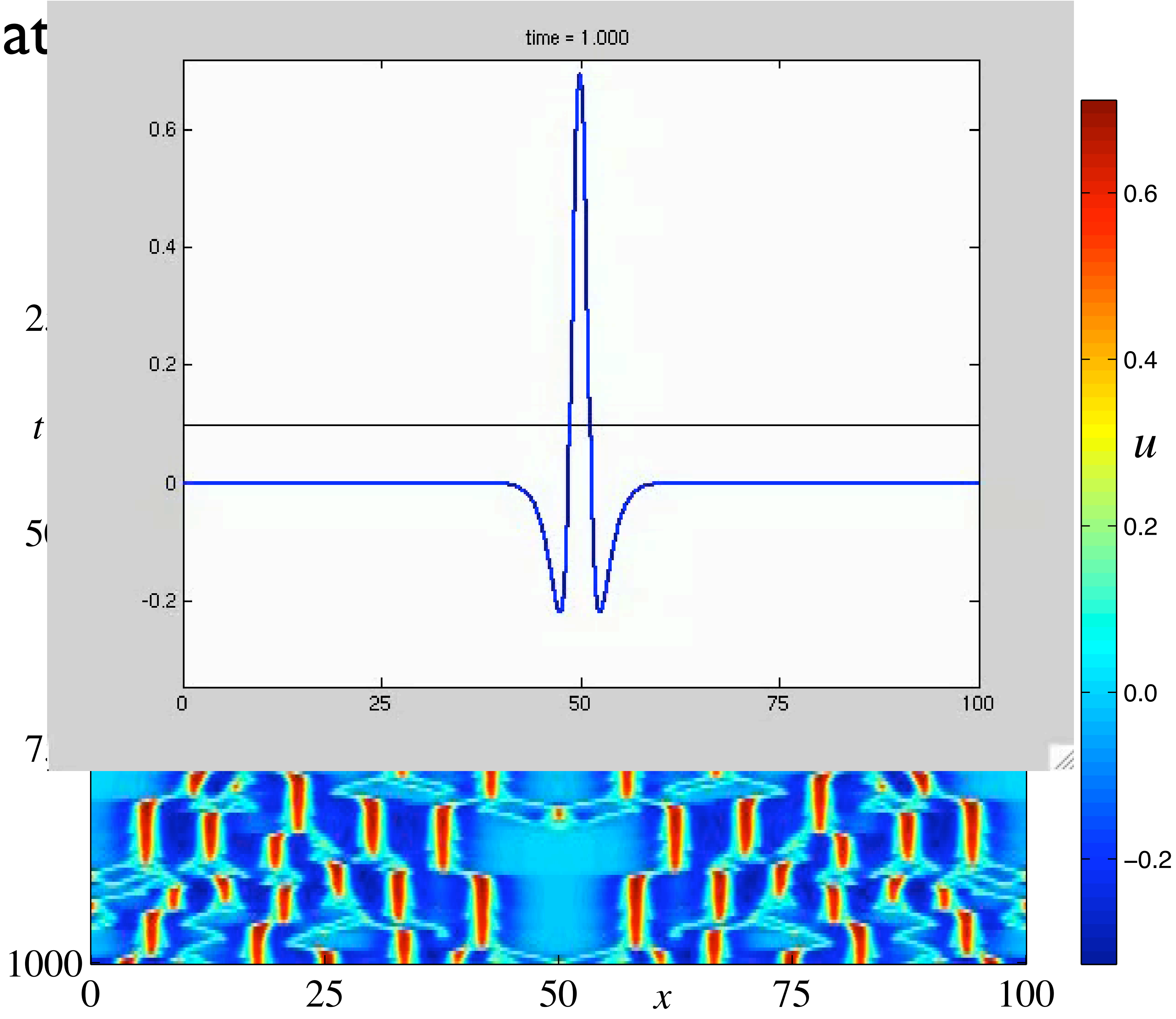

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2D: Dimple Bumps

Complex splitting

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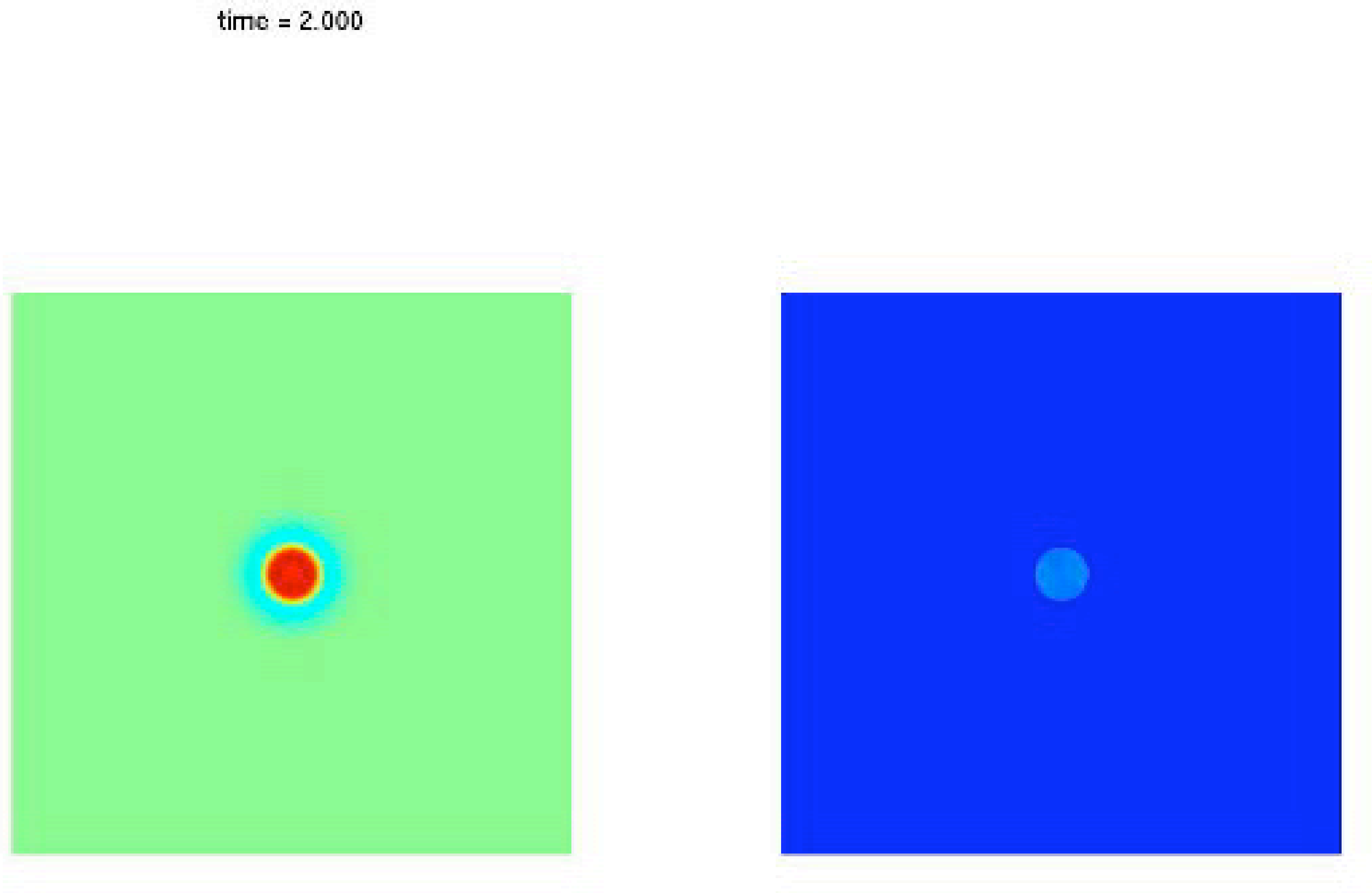
Complex splitting



2D: Collisions

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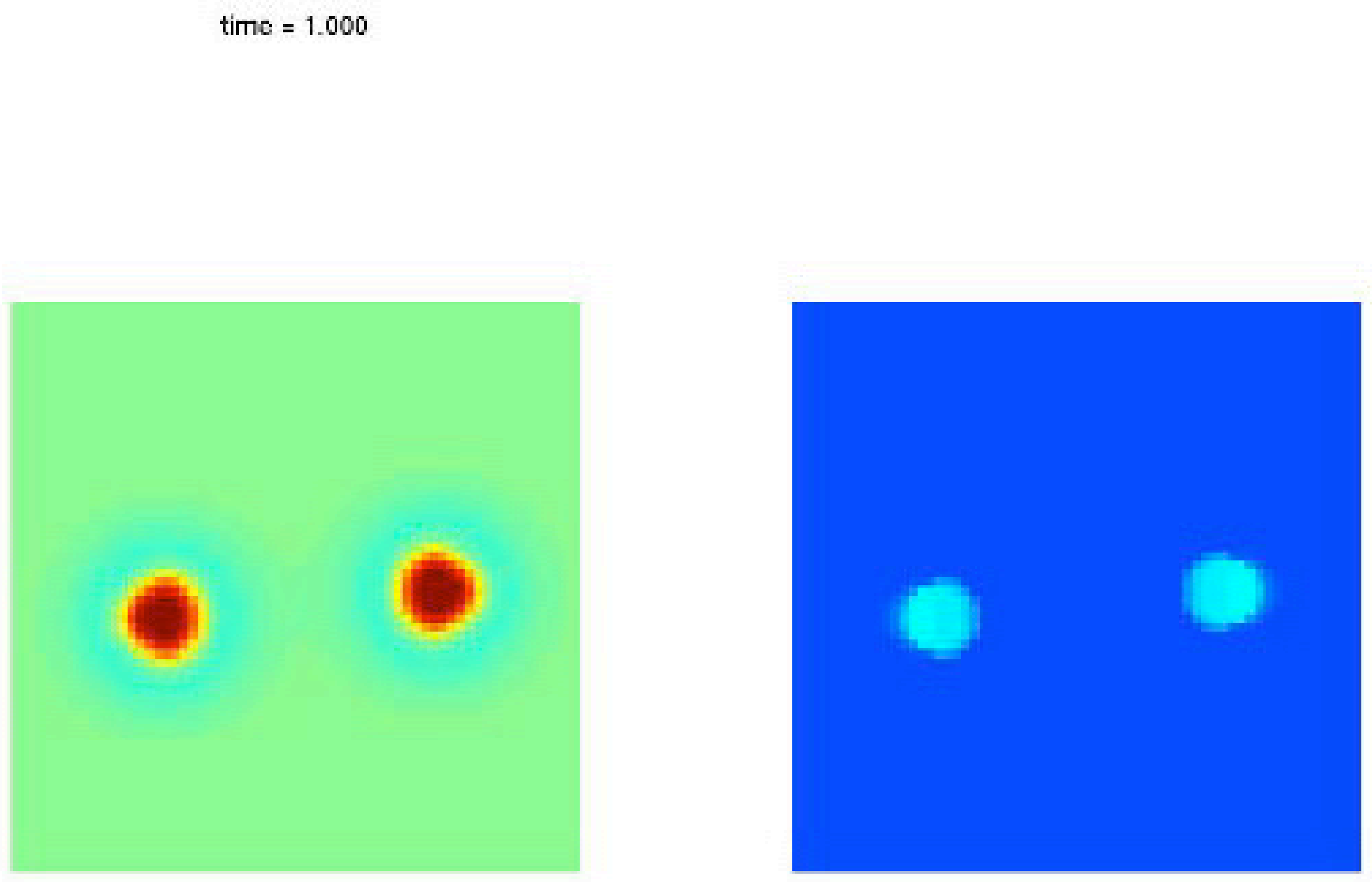
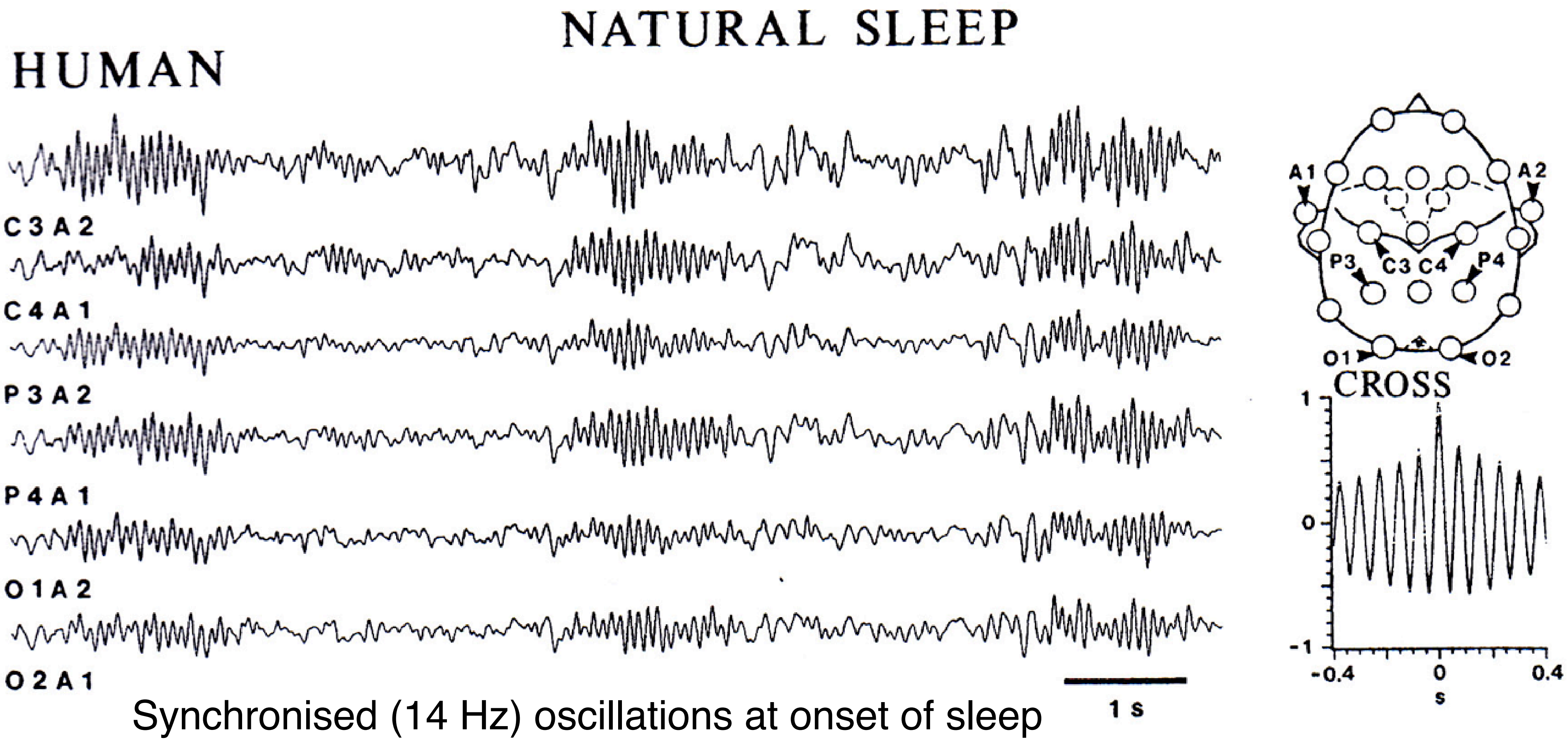
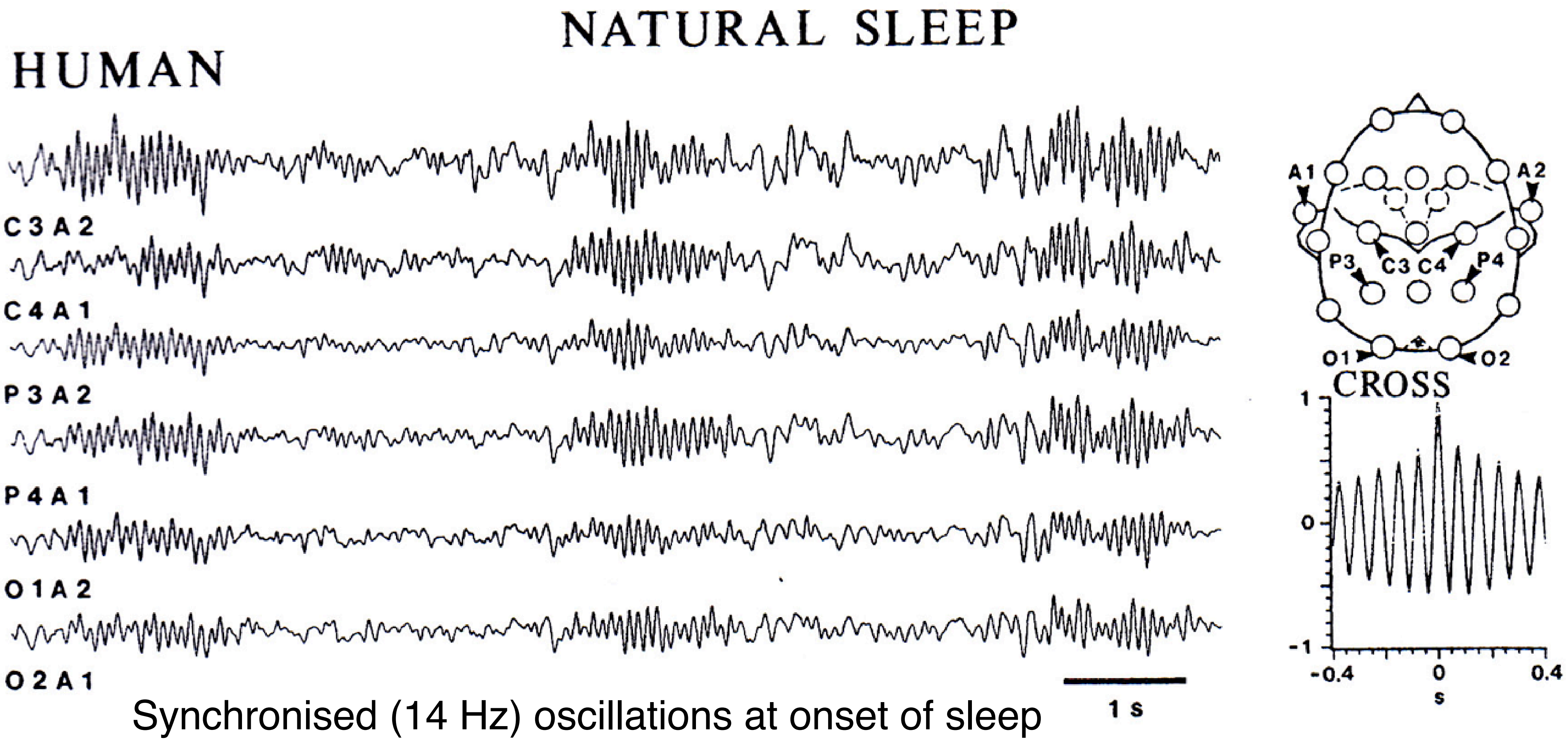


Rhythms and slow ionic currents



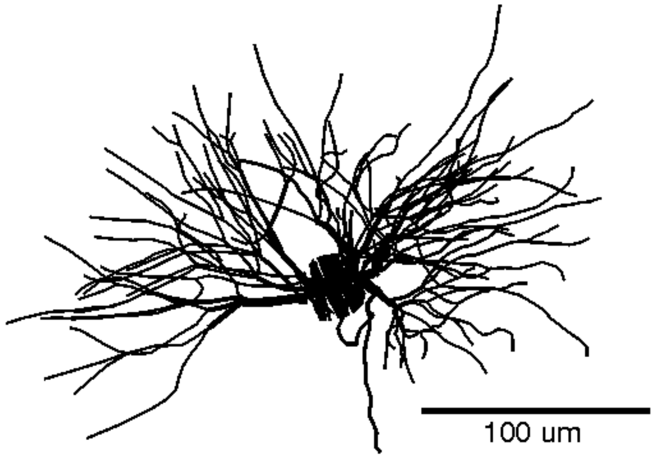
Synchronised (14 Hz) oscillations at onset of sleep

Rhythms and slow ionic currents

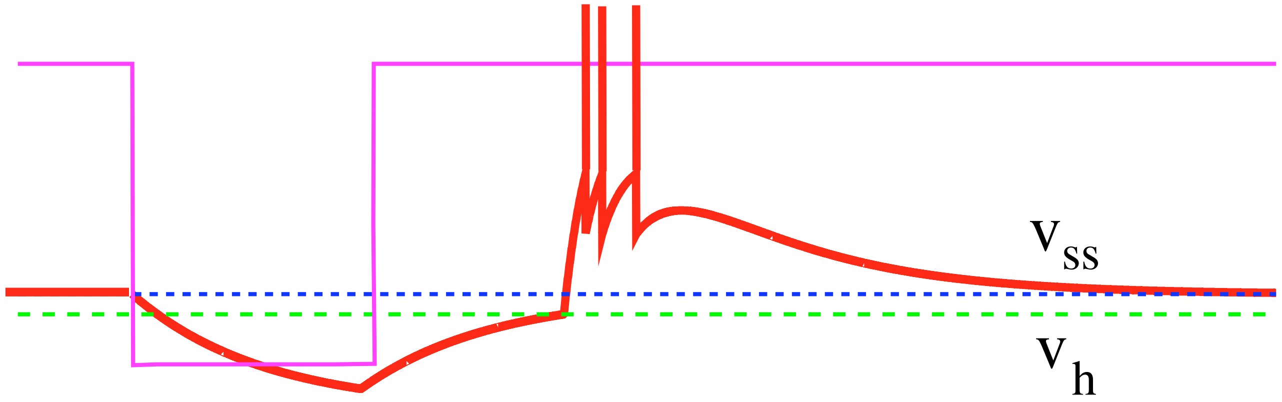


Synchronised (14 Hz) oscillations at onset of sleep

Thalamocortical (TC)



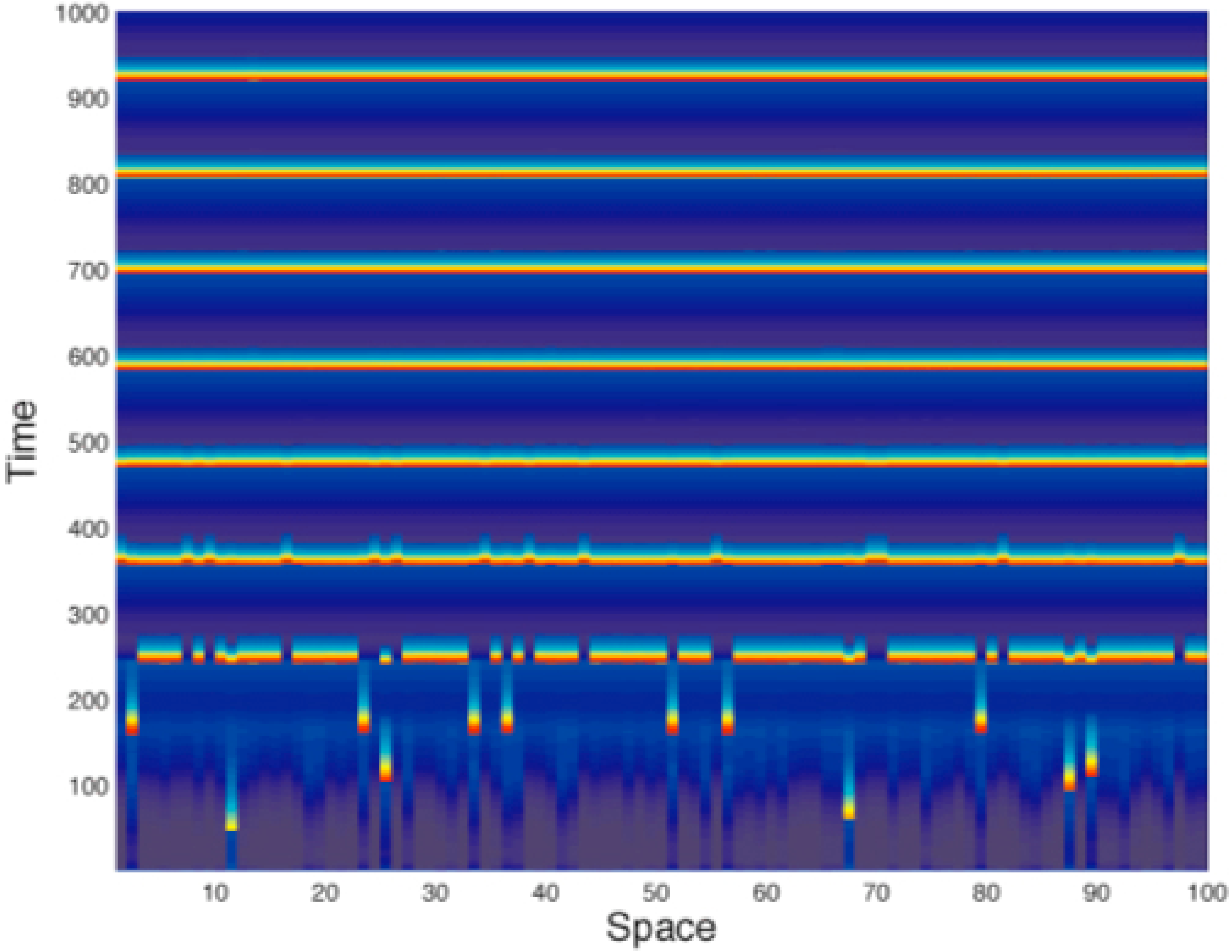
Post-Inhibitory Rebound

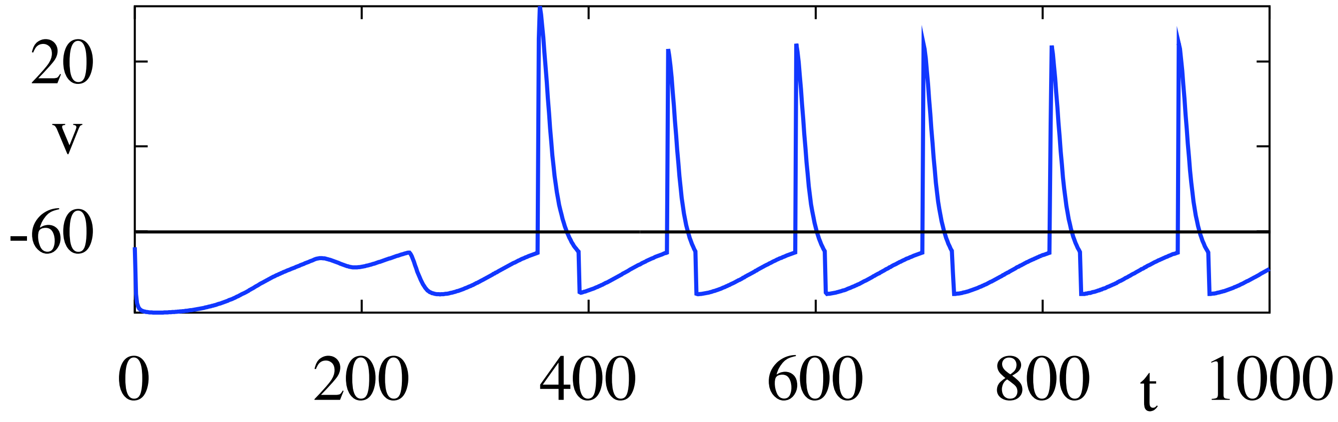


Dynamic response due to a low-threshold Calcium conductance.

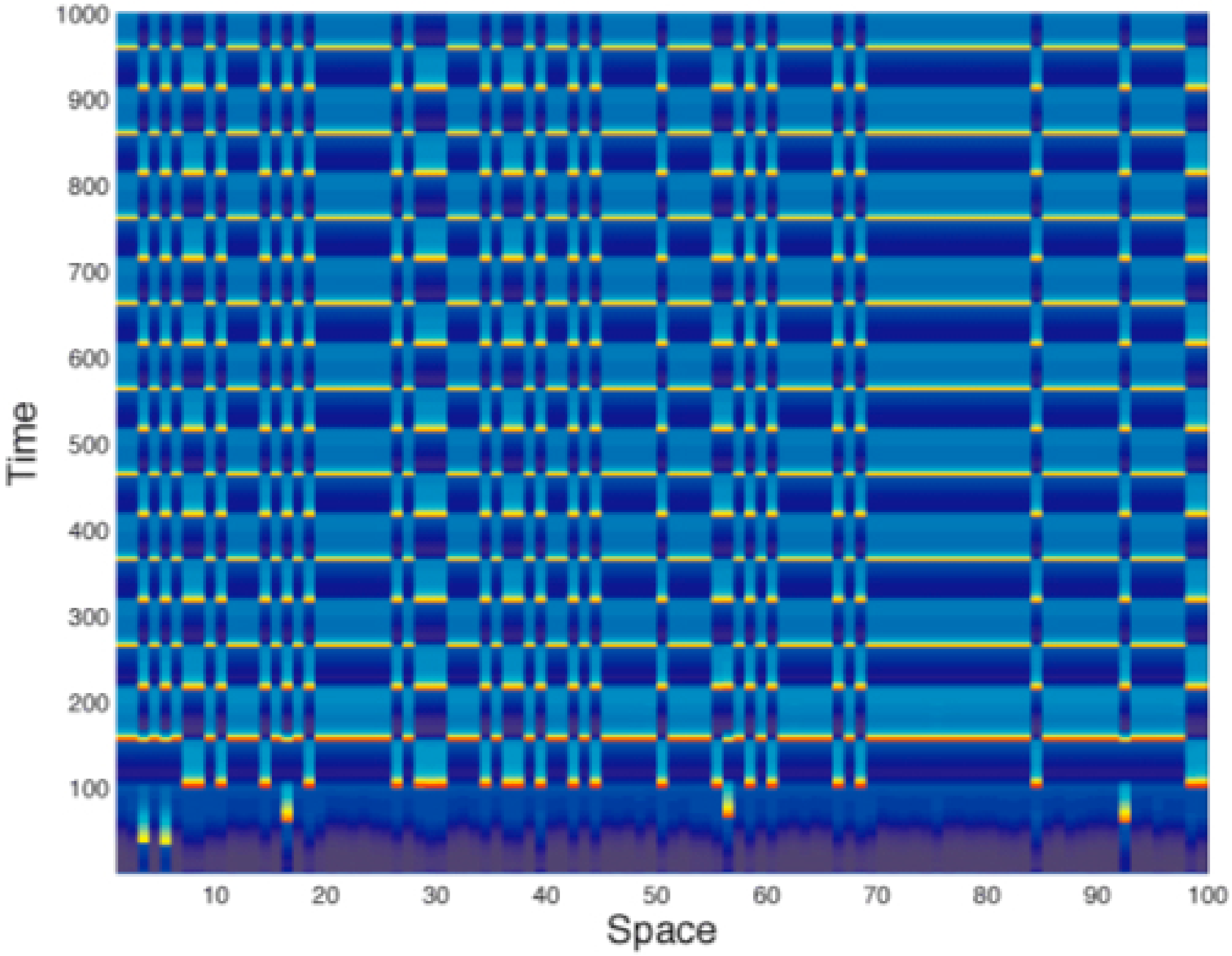
Global inhibitory TC Network

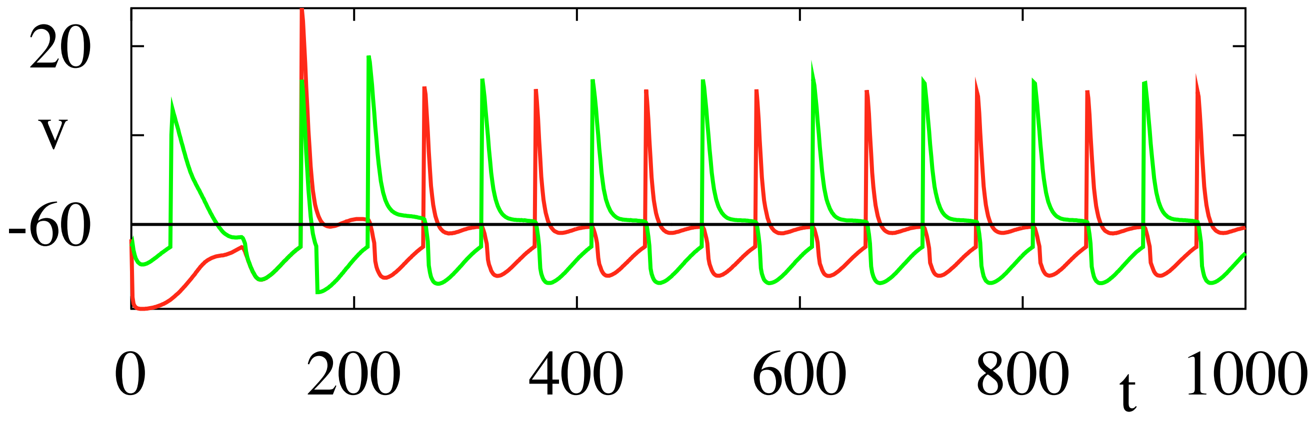
Synchrony for slow synapses





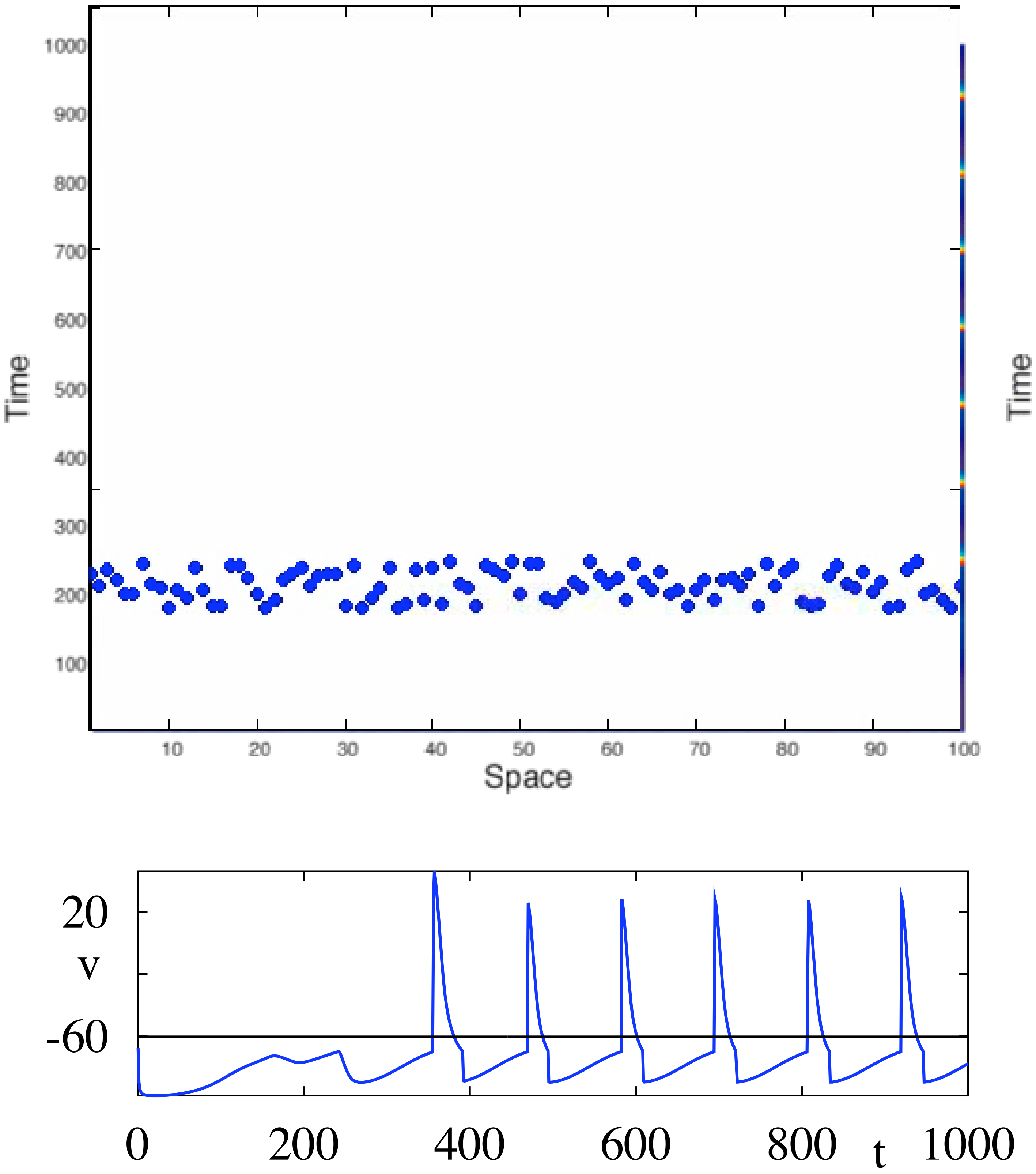
Clustering for faster synapses





Global inhibitory TC Network

Synchrony for slow synapses



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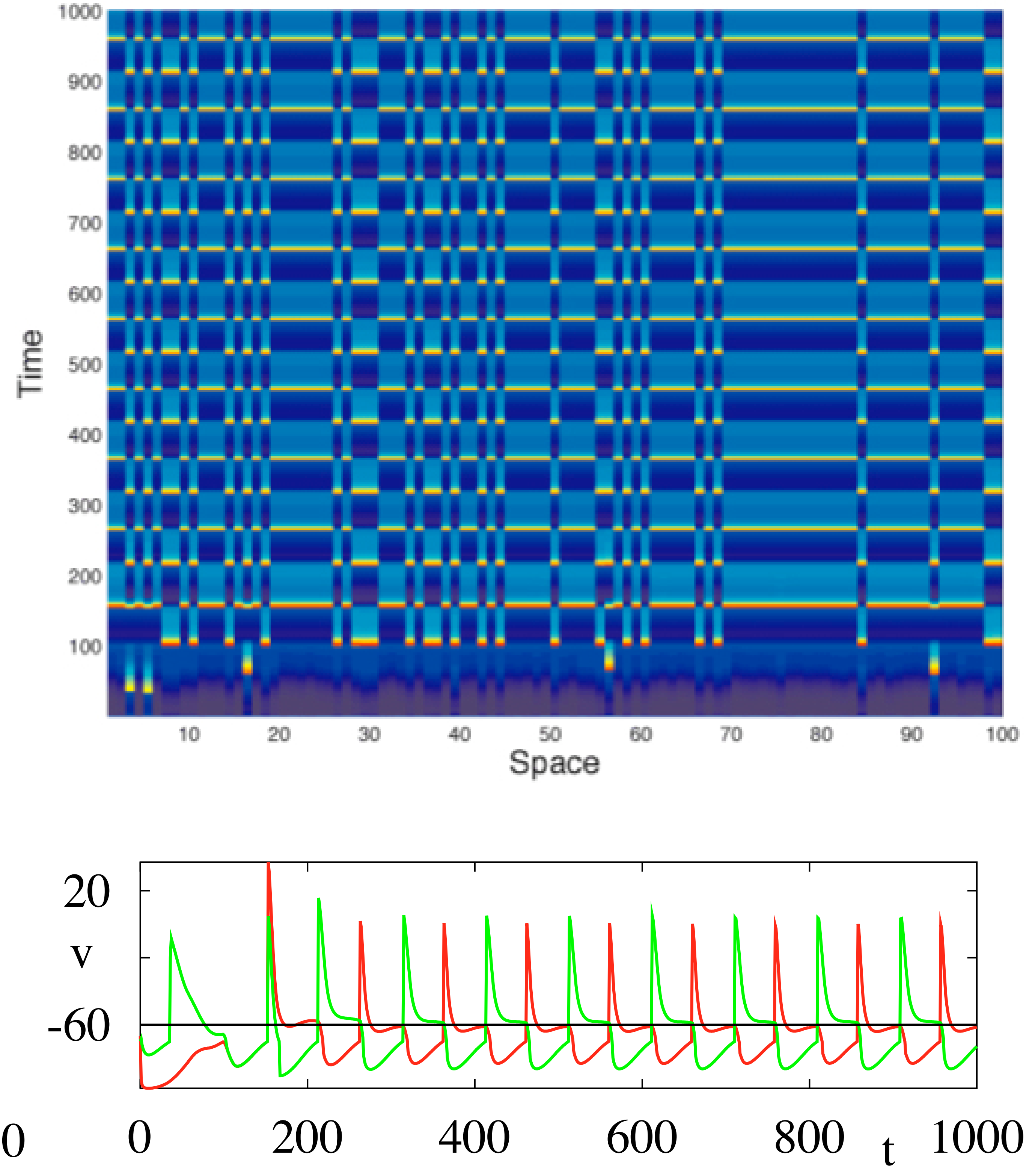
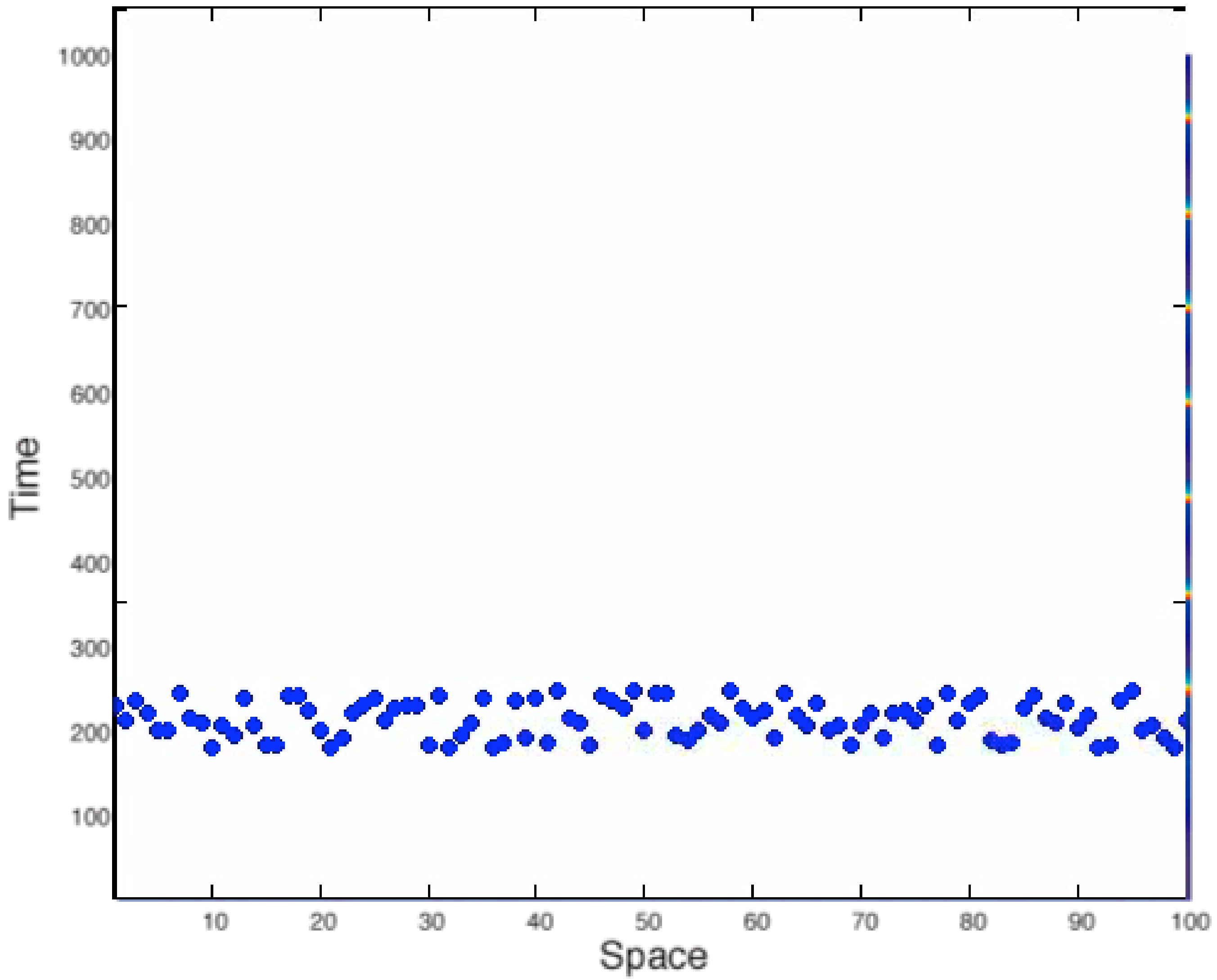
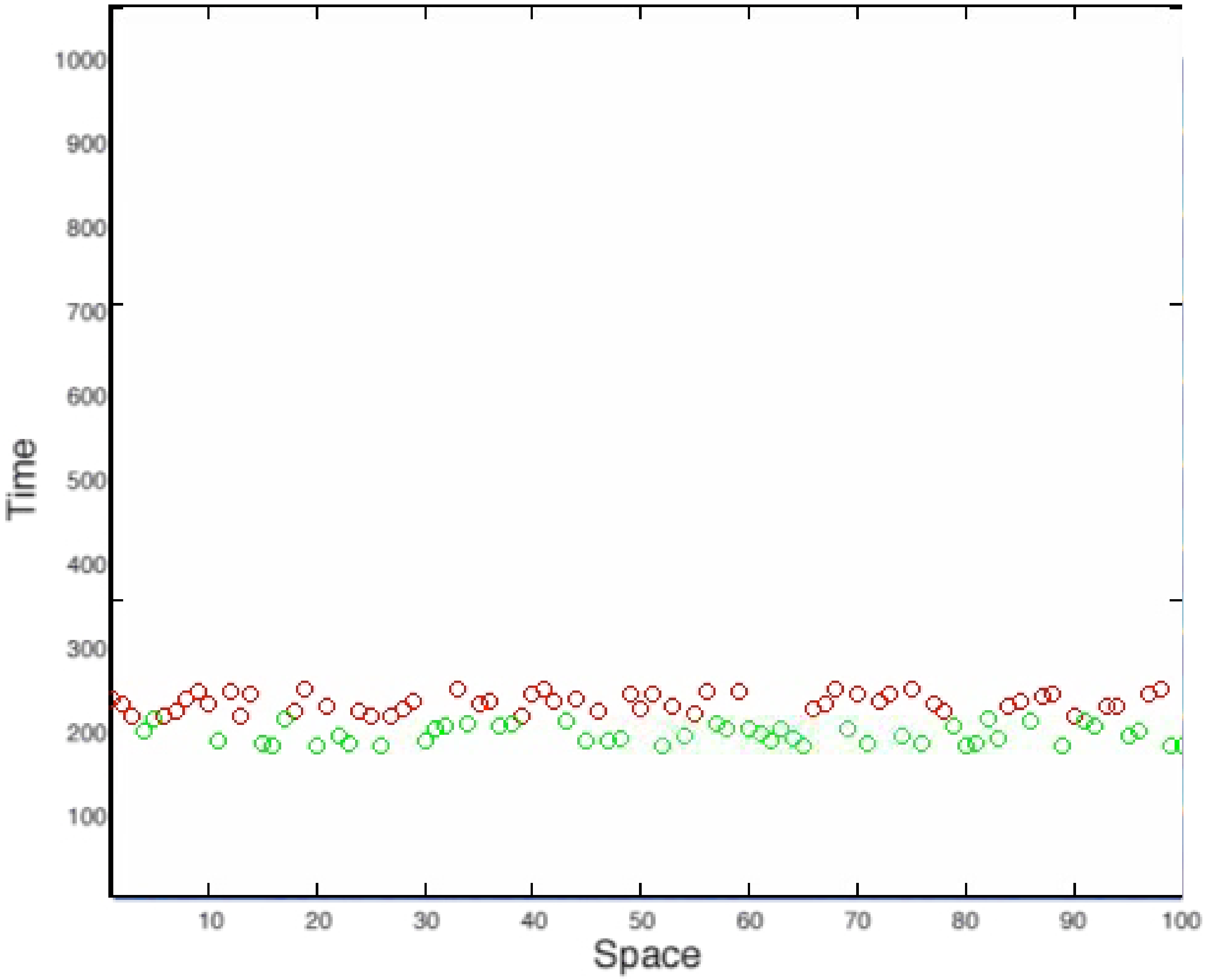


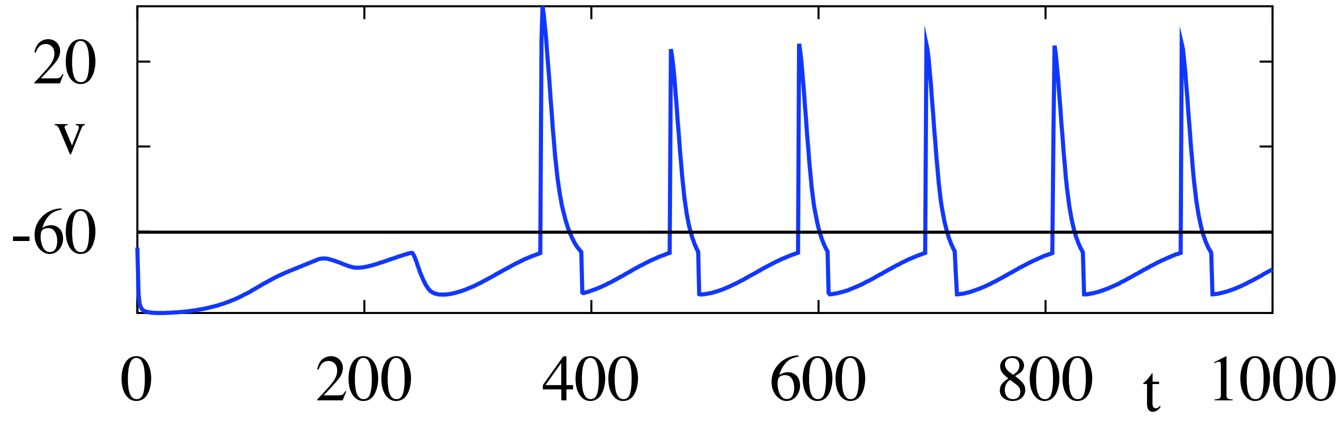
Global inhibitory TC Network

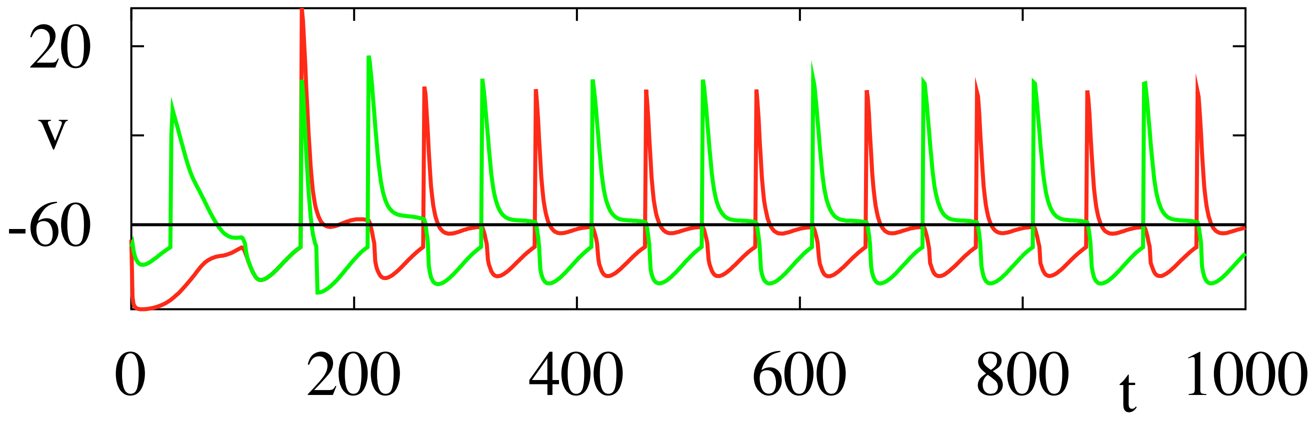
Synchrony for slow synapses

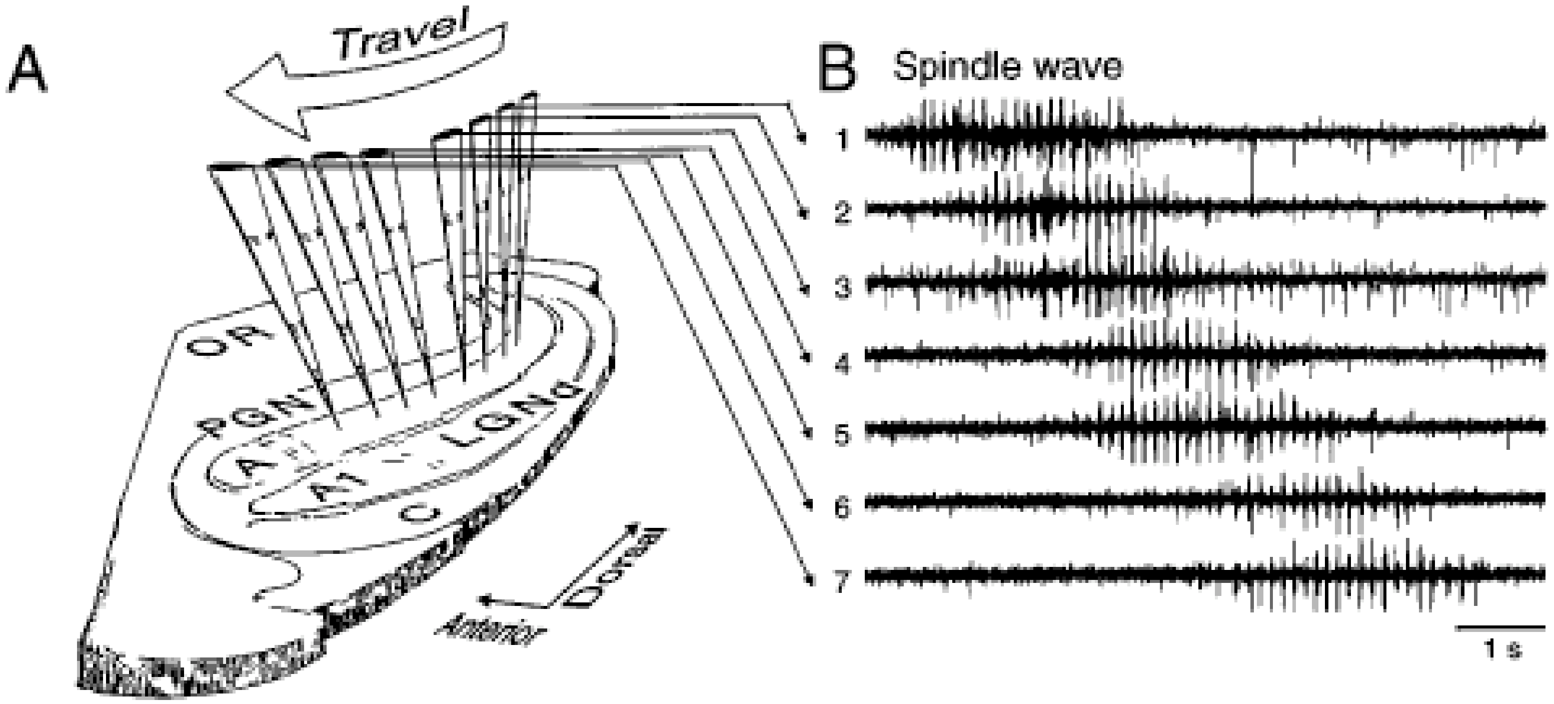
Clustering for faster synapses










A
Travel
OR
PGN
A
A1
LGNd
C
Dorsal
Anterior
B
Spindle wave
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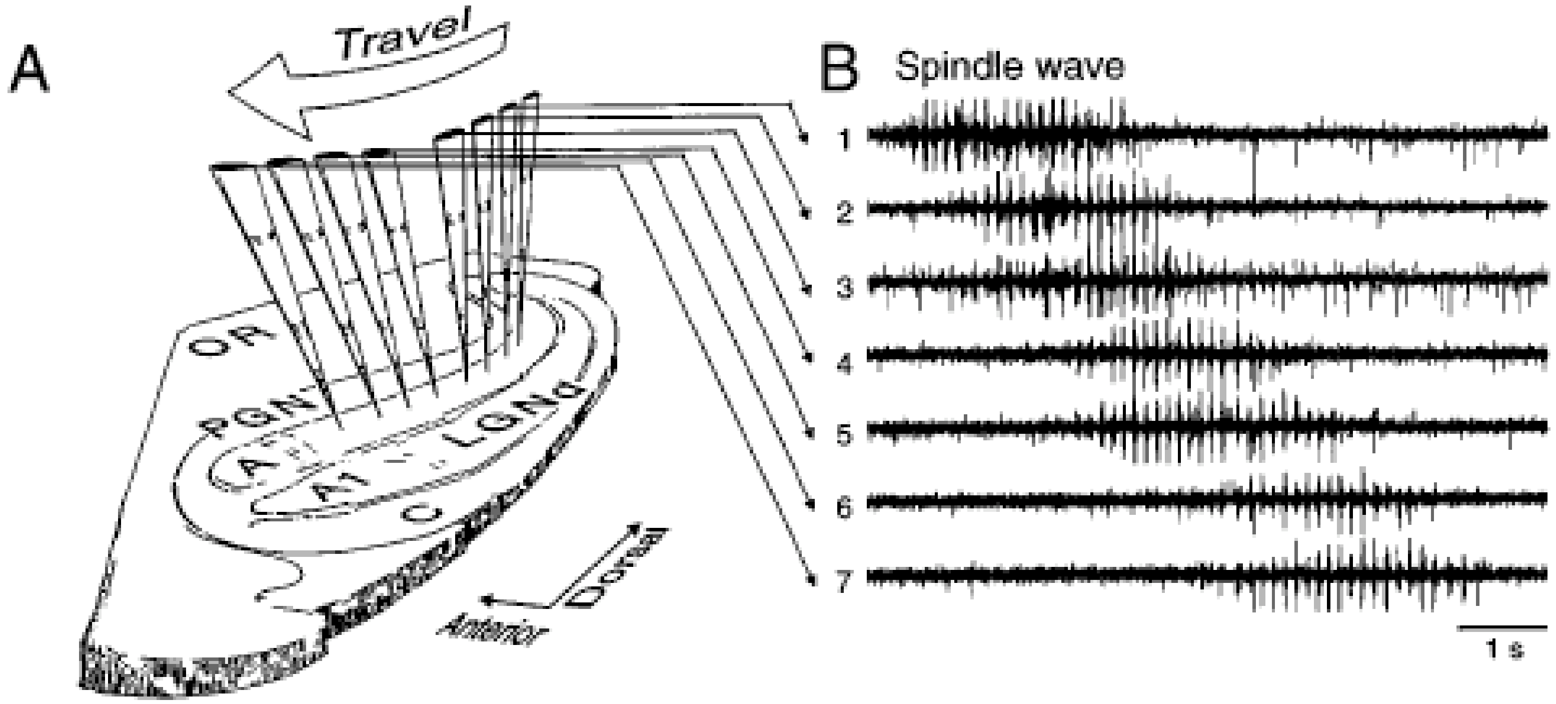
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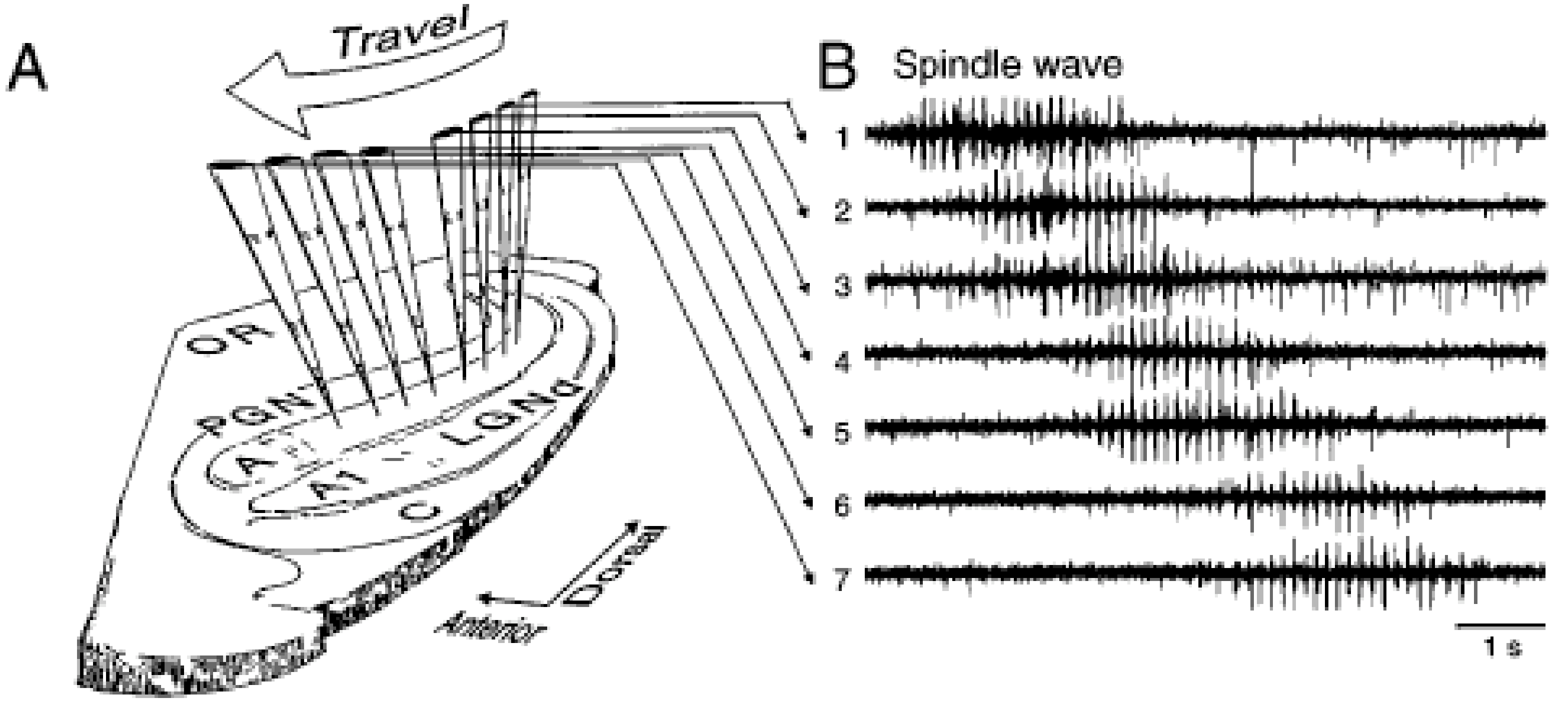
short
range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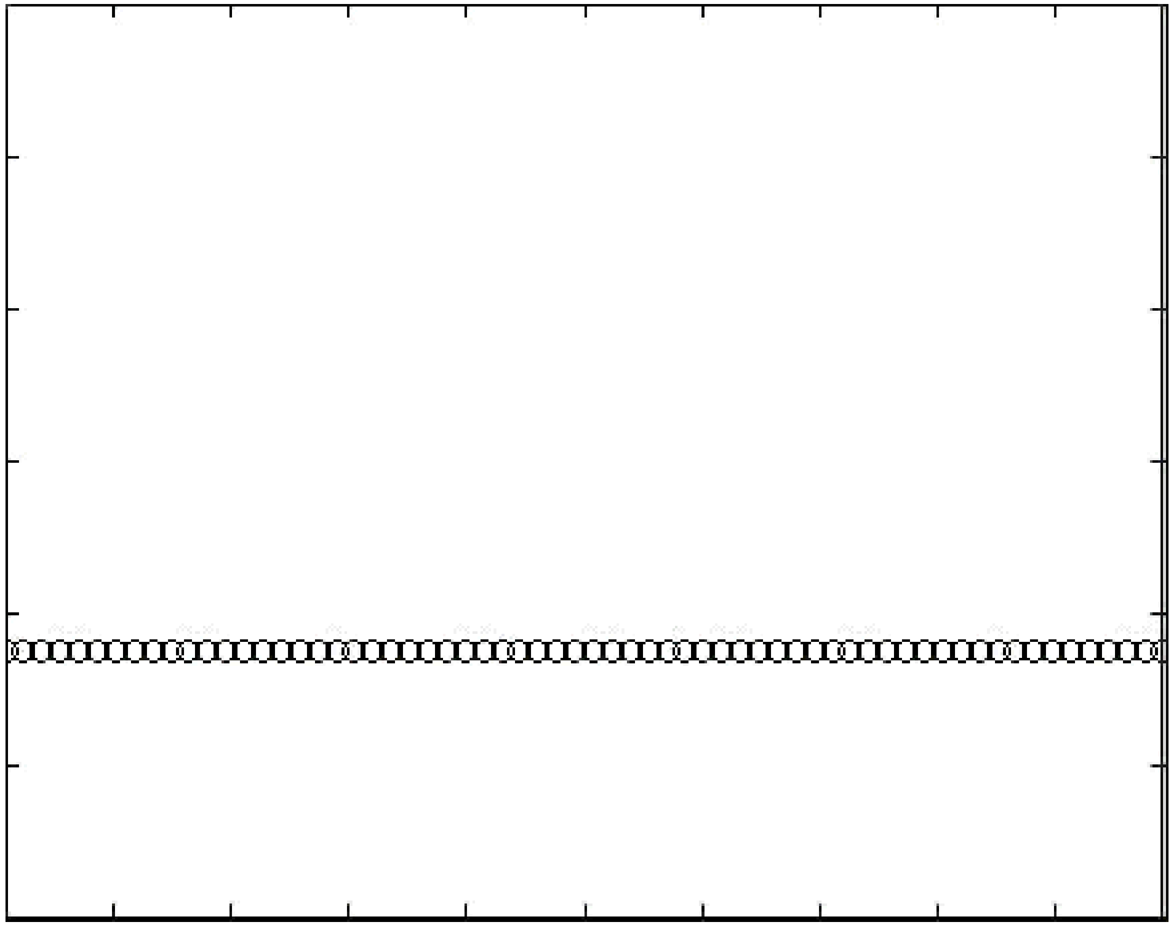

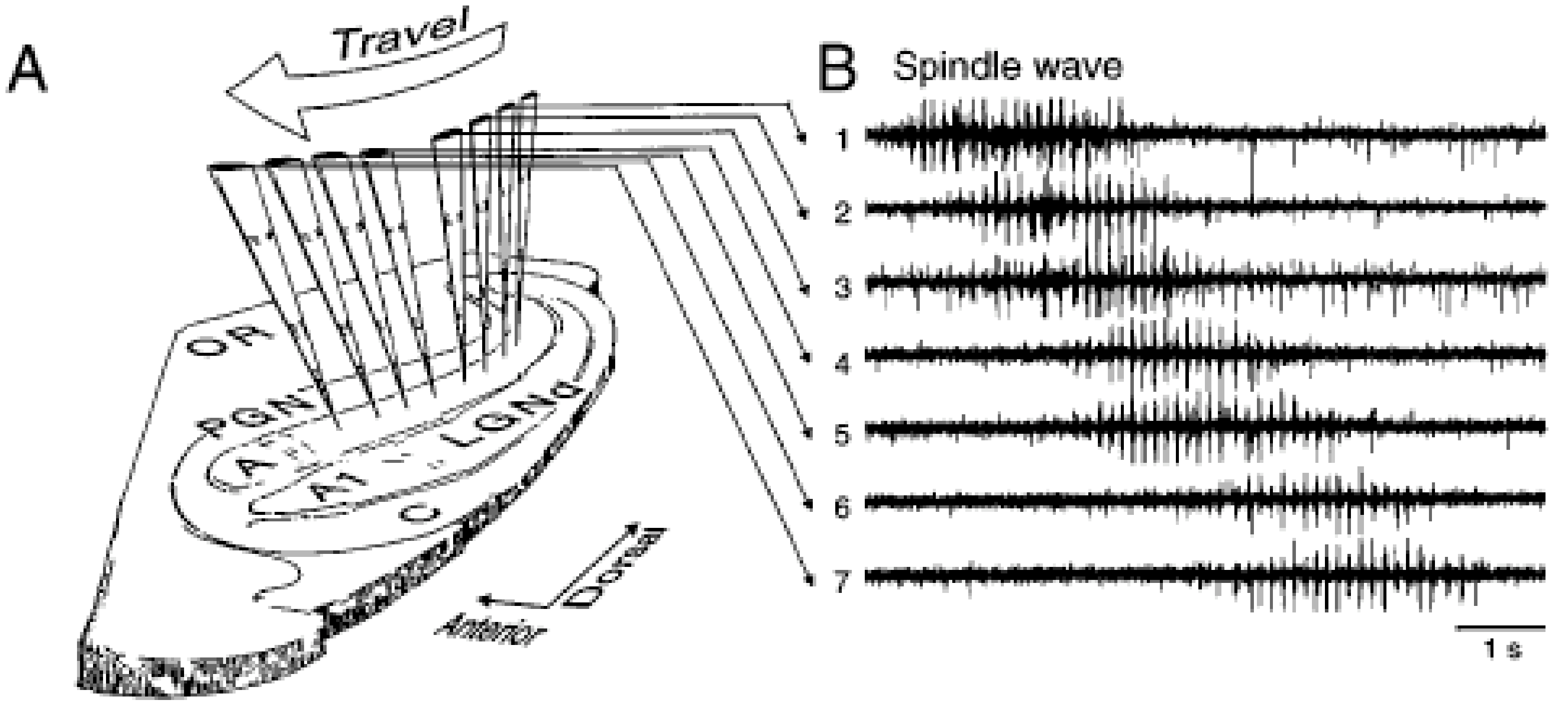

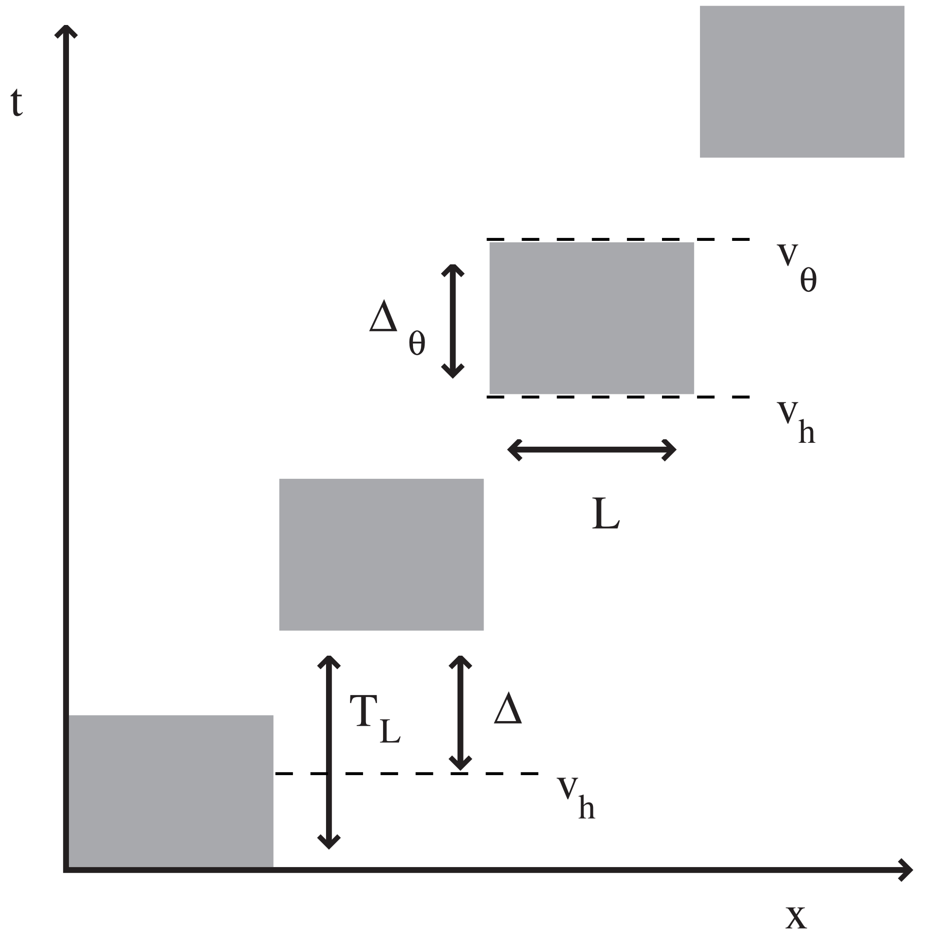

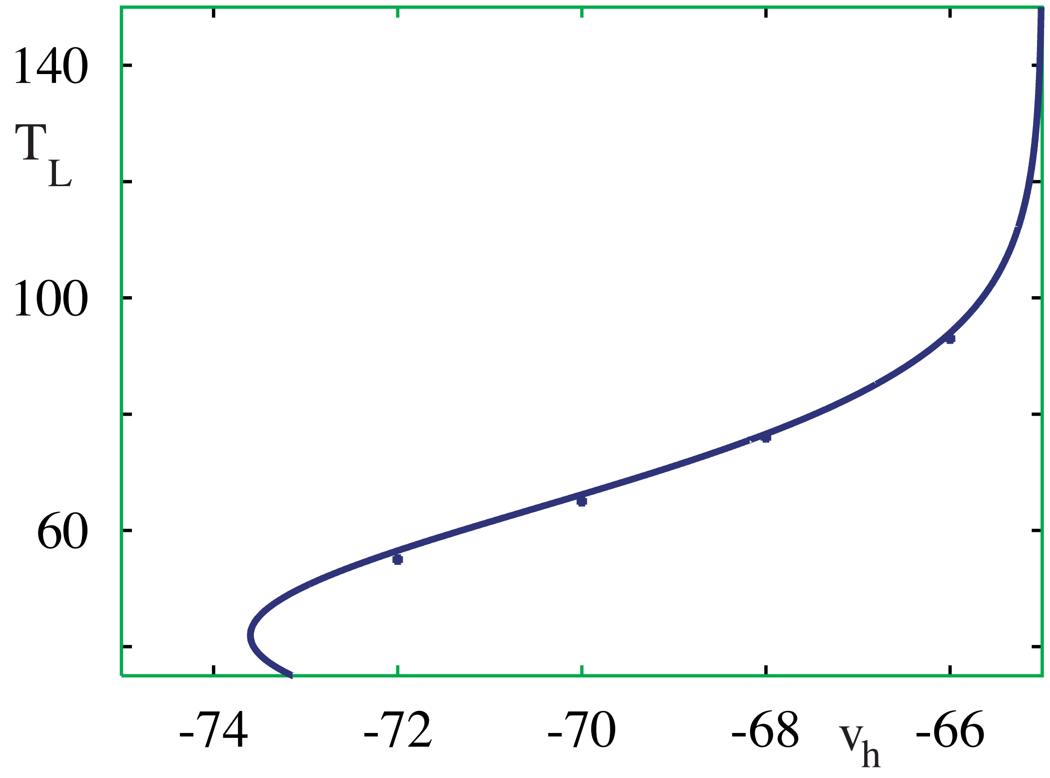

short
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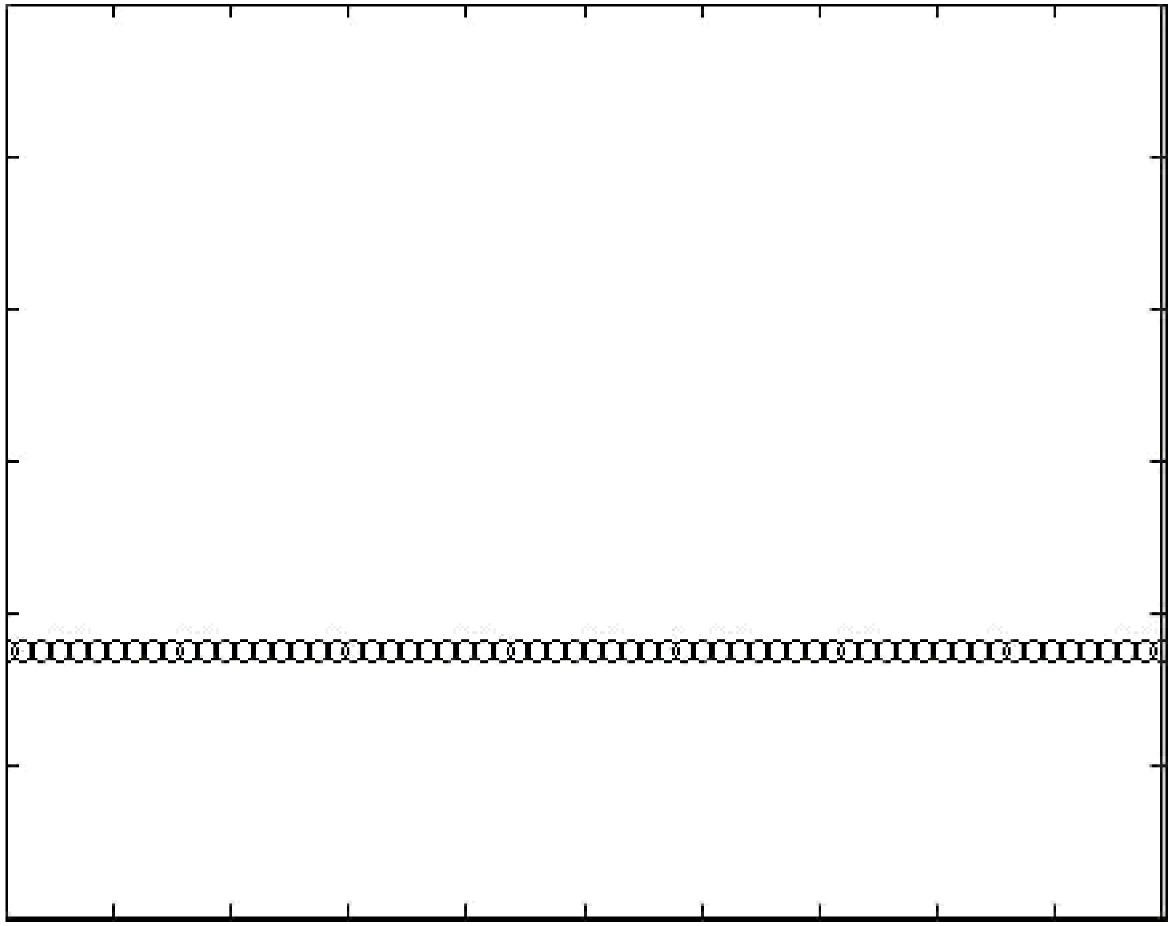
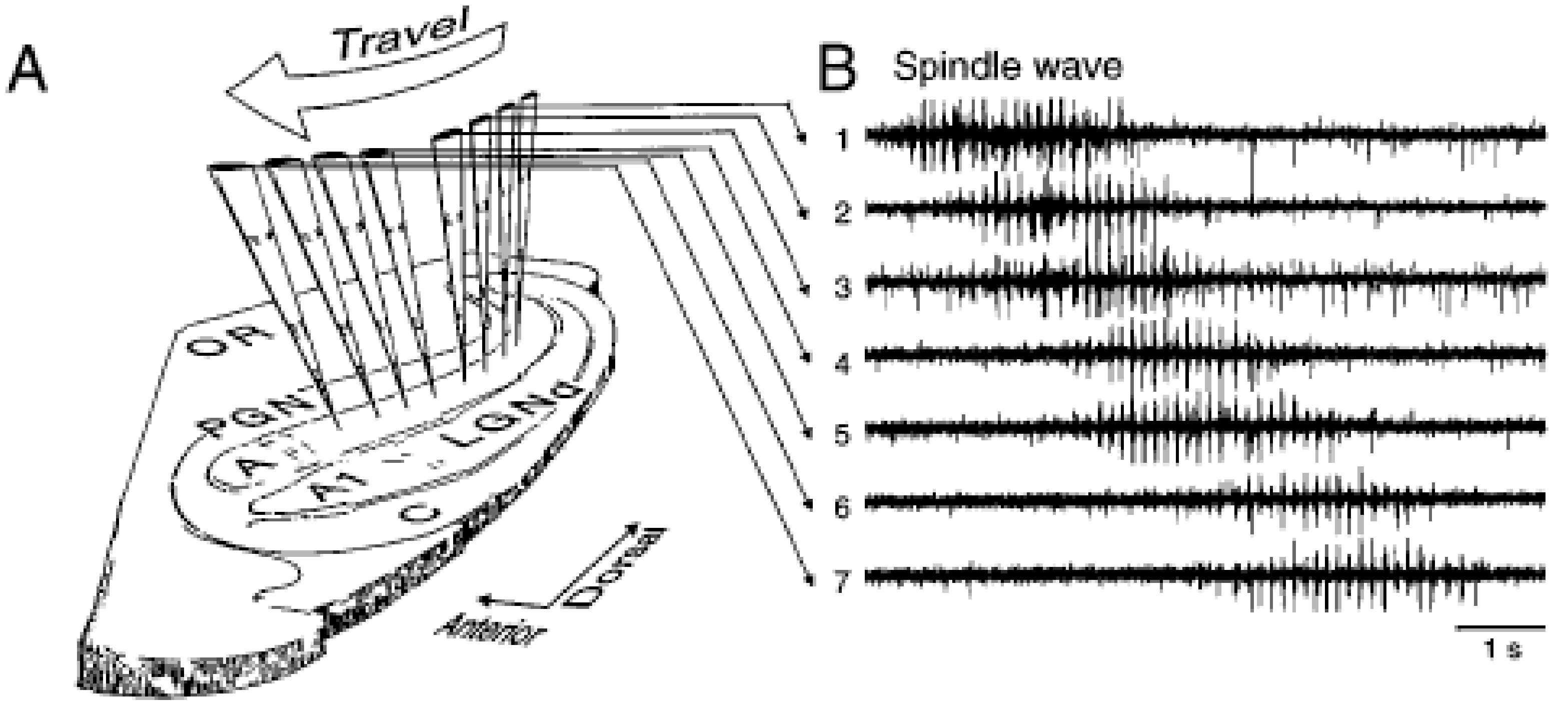
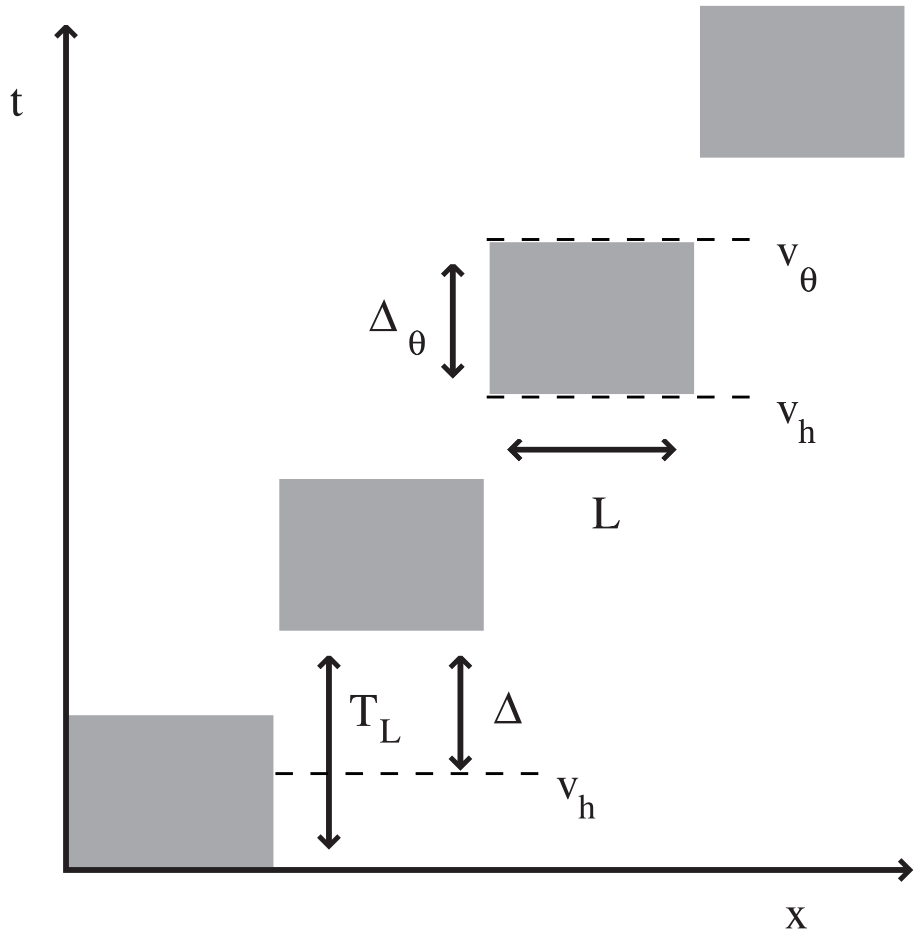
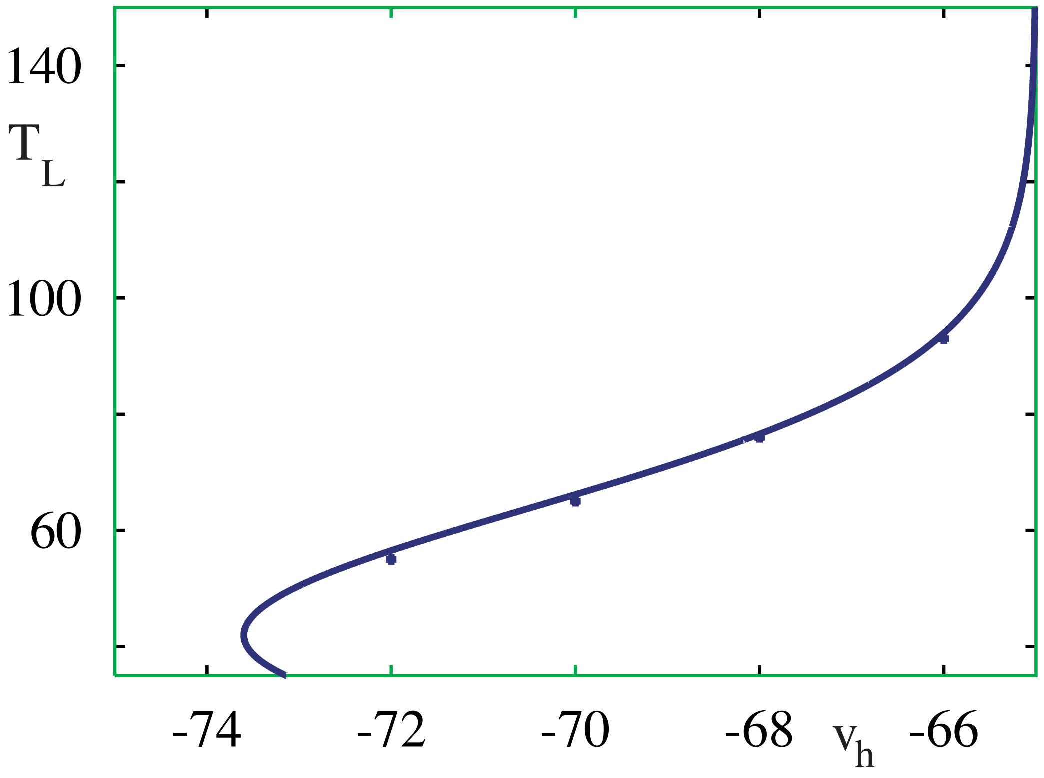
S Coombes, Dynamics of synaptically coupled integrate-and-fire-or-burst neurons, *Physical Review E* **67**, 041910 (2003)









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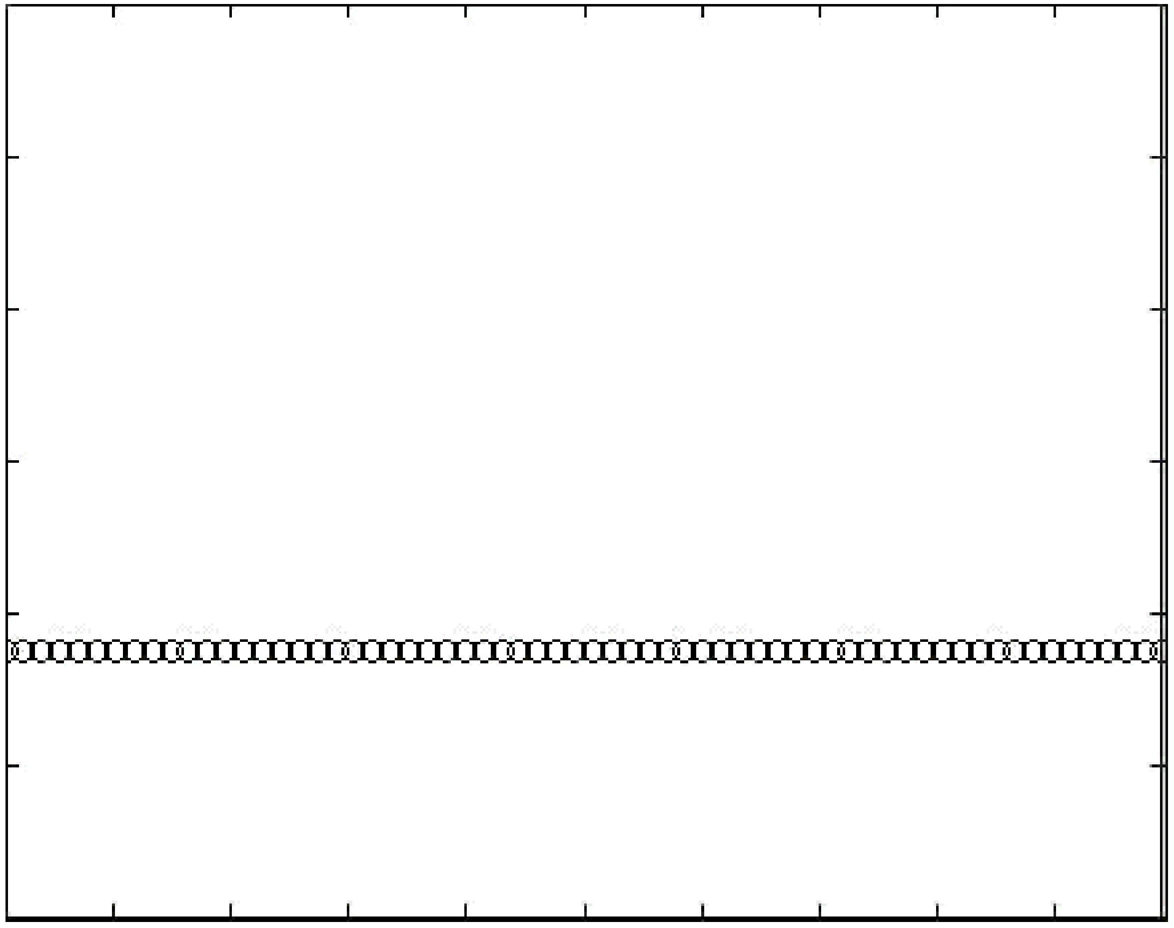
Slow lurching waves in a purely inhibitory network



In collaboration with

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Gabriel Lord (Heriot-Watt)

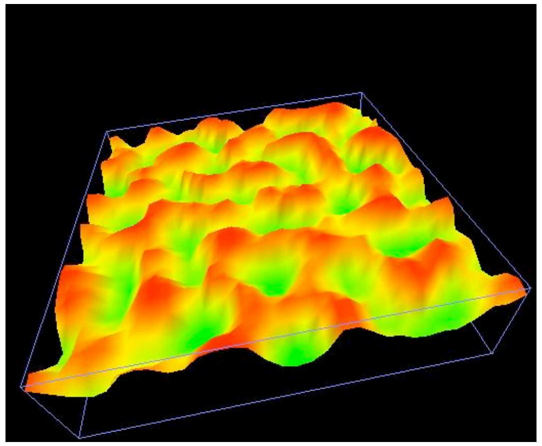
LieJune Shiau (Houston)

David Liley (Melbourne)

EPSRC Engineering and Physical Sciences Research Council

Markus Owen (Notts)

Ingo Bojak (Nijmegen)



Carlo Laing (Massey, NZ)